Redistribution systems theory as a key to reality decoding

Teorie redistribučních systémů jako klíč k dekódování reality

PETR BUDINSKÝ, RADIM VALENČÍK

In any theory development, we may generally distinguish three stages. Its origin is based on an idea enabling to make first small steps. For some time, they remain unconvincing, insufficient, and seem to be just little utilisable. The emerging theory suffers not only from lack of supporters, but also opponents. There are namely not many of those who would notice that something new and prospective is emerging. By entering the second stage, smaller and larger discoveries made within the theory increase faster and faster. They accumulate with a snow-ball effect. The theory proves prospective, consistent, and it opens the space for research that attracts expert supporters. It offers certain style and method adopted by higher number of experts. In the third stage, the space for further discoveries gets gradually exhausted. Some partial elements of the theory are rather fine-tuned. Theory results are however used plentifully in practice, and procedures of their utilisation are continuously improved. The theory has matured, it remains tied up by its own developed paradigm, and it waits until it is preceded by new one.[1]

The fact that a new theory is emerging becomes apparent in the transition period from the first to the second stage. Indications of the fact that it is a prospective theory include:

- Increasing new knowledge and discoveries, relative quick transition from one knowledge level to the next.
- Each new step in the theory development raises great amount of questions. Figuratively speaking – upon opening one door, there are many others awaiting their opening.
- Frequency of questions and possible continuations in theory development lead up to certain scepticism, when we ask whether we are able to defend theoretical issues of the theory in question.
- Various directions of theory distribution start interfering, and the new knowledge gained this way becomes more conclusive, obvious and distinct, concerning both the meaning and the method of use.

The above applies in both the natural sciences area (e.g. theoretic mechanics, molecular kinetic theory of gases, elementary particles theory, etc. were born like this) and in case of social sciences (theory of use, Keynesian theory of macroeconomics, etc.).

1 *Compare e.g. Kuhn (1982).*

Six months after we published the English version of the article called Nash Equilibrium in Redistribution Systems (calculation, meaning, and utilisation) in ACTA VŠFS[2] magazine in February 2008, we acquired much more knowledge in this area in a relatively short time, and walked through another part of the path leading to recognition of what goes on in social systems. It seems to us that we are at the beginning of a prospective theoretic concept.

Let's remember that under the term "redistribution system" we understood such system, in which certain movement of compensations as against performance of individual players takes place, resulting in its reduced efficiency. The cause of compensation movement as against performance of individual players is considered development of alliances inside the system, when players in the alliance controlling the system are favoured at the expense of players who are not part of this alliance. In other words, we can see the social system as a system, which various lobbies win their recognition in, when those controlling the system increase their compensations at the expense of those who are not part of these lobbies, which at the same time results in reduced efficiency of the entire system. The area of application of the redistribution systems theory is very broad. More specifically – we would barely find a social system that is not more or less influenced or at least threatened by what the redistribution systems theory tries to express using its mathematical apparatus. In the article published in February 2008, we presented a model of the elementary redistribution system, showed some of its features, and determined and calculated discriminating equilibrium and Nash equilibrium3. We at the same time indicated possibilities of further model extension and practical importance of modelling social systems using the apparatus of the redistribution systems theory.

1 How we recognise the reality through abstract models

We can recognise the reality in various ways. In some cases, we identify, designate, and classify what we encounter. We find the general and search for the causative or other connections. This way, we may proceed also in case of the analysis of various social systems. We may for example find that various alliances are formed not only inside these systems but also between them. These alliances are often hidden and decide on how the situation inside individual social systems would develop. When analysing procedures based on monitoring and experience only, we can find out soon that there are too many various connections or links as well as their intermediations. The possibility to foresee the development in individual systems is thus limited.

Another possibility is to use an abstract model. The process of reality recognition then looks like a process of "deciphering" or "decoding" of that area, when the key or the code is the model itself. The experience from reality recognition says that the condition of success consists in revealing what is very simple, i.e. really elementary. By expanding, enlarging, supplementing, generalising (extending) the preconditions, incl. hidden ones, the model then gets an image when it enables to capture the richness of the reality, and it is capable to describe it more comprehensively. It concurrently shows that it is necessary to define

2 Budinský – Valenčík (2008)

3 Nash equilibrium was interpreted mainly according to Carmichael (2005).

(capture) root definitions again and again, and more precisely. Their precise approach is just the key to revealing hidden preconditions and thus also to model development.
In doing so, it is important to find the simplest way. The simplest within the sense of intuition, since there is no formal regulation that would express what the simplest is and how to find it. If we then assume the simplest (the key), the reality starts opening in front of us, and we can start reading it.

With respect to importance of the principle to use the simplest model to read the reality, we will dedicate couple of comments to it. The most illustrative example of finding such key to read the reality was probably discovery of the law of inertia based on abstraction of the uniform linear motion. It is sometimes ascribed to Galileo, but it was rather discovered by his followers. It seems that there is nothing simpler then to express motion of an object, which is not influenced by anything, by equation $s = v*t$ (the stroke s is directly proportional to the velocity v and the time t). This is thus a base of the well known formulation that the object remains standstill or in uniform linear motion, if no power applies to it.

For more then two thousand years, there was however a concept of objects motion based on the authority of Aristotle, saying that objects need constantly a power for their motion. A new concept based on abstraction of the uniform linear motion was emerging (gaining grounds as against established ideas and stereotypes) with great difficulties.

This concept then became a key (code) to read (decode) the reality. It for example raises a question – What if the object is affected by power? The second Newton's law then results from the answer - generalisation of the motion to uniformly accelerated one. Another example of extension is the power generated by mutual interaction of two objects (the law of action and reaction, or the third Newton's law). This was just a small step away from the molecular kinetic theory of gasses and liquids, etc.

It is important that each step leading to extension of the original model as well as the next step extending models generated by its extension was always managed in the direction of the simplest expression of the overlap.

Also the transition from the classical mechanics to the new ones, based on the relativity principle, was distinguished by the fact that the overlap featured (from the intuitive point of view) the simplest or the most economy solution.

We may imagine each step of word decoding as a limit-simple extension of the original (also originally limit-simplest) model. It is among others testified by some contributions published in the book My Einstein (the original was published in 2006). These are mostly contributions of top physicists who, with a dispassionate point of view and deep understanding, comment on what Einstein actually came up with. We can mention one of them. It is adumbrated by its title "Albert Einstein: Scientific "reactionary": *"Einstein resolved this discrepancy as a twenty six year old young man in 1905. He later reported that as soon as he realised suspicious nature of Newton's axiom of absolute time, he was able to find a method of modifying Newton's mechanics within six weeks, to be in accordance with Maxwell's equations. Since Einstein's mechanics and its best-known definition* $E = mc^2$ *have brought revolutionary consequences, we rarely realise that Einstein's innovation was basically very conserva-*

tive. His modification of basic physical equations valid at that time was just minimal… When Einstein introduced the basic velocity limit, i.e. light velocity, to the mechanics of objects, it was apparent that the Newton's gravitation theory must also be modified, since it calculated upon unlimited velocity to overcome gravity effects. Newton's law of gravity does not contain any limitation of velocity; gravity effects of rock motion on the Earth should in principle become evident anywhere in the universe. By 1917, Einstein created successfully a new gravity theory... His gravity theory, called regularly the general relativity, is often considered a revolutionary change in looking at the gravity, since Newton's gravity is a power, while in Einstein's concept it is a space and time curvature. In spite of that, general relativity was also actually just a conservative modification of the existing Newton's gravity theory… Einstein's gravity theory enabled curvature of both, time and space, and showed that those curvatures are mutually related. What could be more natural?... Also Einstein's discovery of quantum mechanics was again a conservative modification – conservative in terms of preserving classical structure of Newton's physics…" (Tipler 2007, p. 82-84.)

The fact that Einstein's modifications of the original (Newton's) model were "reactionary" or "conservative" means nothing else but they were the simplest (limit-wise) extension. Tipler's erudite interpretation enables to understand well what this simplicity consisted in. And in terms of methodology, it is instructive for any work with a model that strives for (if possible) general validity in that area of reality. Both, the first model and each (emphasis is on the word "each") extension that is to be beneficial in discovering of what is going on must meet the criterion of intuitive (hard to define, but apparent by internal feeling) limit simplicity. This guide was and still is important also in our case. It was at the same time confirmed that the above process can be used not only in the area of natural but also social sciences. And as we show you thereinafter, we can get very far using this method.

2 Practical and theoretical importance of identification and calculation of discriminating and Nash equilibrium in the elementary redistribution system

The model of the elementary redistribution system is conceived to include the most important and also the simplest we can encounter in this area. Its logic consists of three players (A, B, C), each of them with the same influence on the system, any two players may form an alliance, players' performance is acknowledged by small natural numbers (e.g. 6:4:2). We can then display players' compensation in a 3D space, when each coordinate axis corresponds with the compensation to any player. The impact of reduced system efficiency due to compensation departure from players' performance can be expressed by redistribution equation, e.g.:

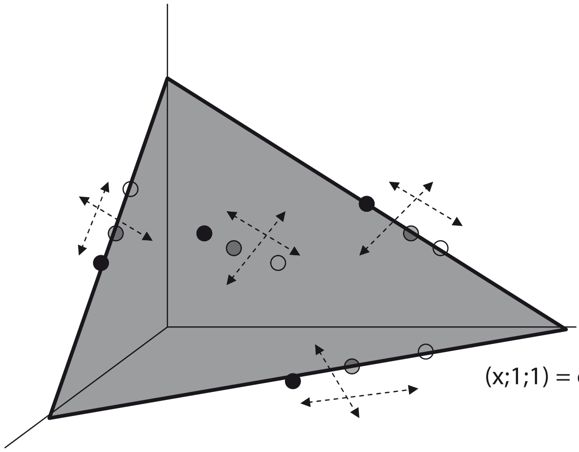
$x + y + z = 12 - \eta.R(x - 6, y - 4, z - 2)$

where $x + y + z$ is a summary of compensations to individual players; 12 is the maximum compensation that could be distributed, if the performance of the redistribution system is maximal, which means that no redistribution would occur, and compensations would be distributed according to the performance; η is a coefficient of performance drop; $R(x - 6, y - 4, z - 2)$ is a function of distance of distributing actual compensations from compensations according to players' performance.

With respect to the fact that the redistribution equation links three variables (compensations of individual players) by single dependency, all possible redistributions may be displayed in the redistribution area in a 3D space. If we assume that the lowest compensation for each player is equal to 1, we can then form an idea on the arrangement of individual types of equilibrium in the redistribution area as follows:

Diagram 1: Various types of equilibrium in the elementary redistribution system:

(1;y;1) = compensation of the player B



(x;1;1) = compensation of the player A

(1;1;z) = compensation of the player C

Where:

- The triangle determines a simplified image of the redistribution area, i.e. s set of all possible redistributions.
- Thin lines of the coordinate represent compensations of individual players.
- ○ Is the point corresponding with the distribution of compensations according to players' performance, in case of both, discrimination equilibrium (if on the border line) and the distribution according to the performance of all players (if inside the redistribution area).
- ● Is the point corresponding with discrimination equilibrium (if on the border line), or Nash equilibrium (if inside the redistribution area).
- ● Is the point corresponding with equalitarian distribution of compensations in case of both, discrimination equilibrium (if on the border line) and the distribution according to the performance of all players (if inside the redistribution area).
- Arrows indicate possible movements of discrimination equilibrium and Nash equilibrium in case of external influences on the redistribution system.

Graphic display of individual equilibrium types and their movements enables to form an idea of what is going on due to various influences acting on any system (that is also always a redistribution system). Everyone of us estimates what would happen. The ability of our estimate is considerably individual. We have available various information, our own experience, we include the fact that "wish is the father to the thought" in our estimates in various ways, etc. General scheme may however significantly consolidate such our individual ability.

In doing so, it is very important to mention the following: As soon as we encounter that something develops differently than it should, it is a signal that there is a new influence or factor, which we did not envisage, e.g. a network link of any game participant, or the fact that he/she is a member of certain hidden cross alliance.

It applies in general that the role of social networks influencing the results of negotiations inside individual and relatively independent systems (of redistribution system type) is very strong and growing. It is appropriate to take it into account. Model confrontation as a visual image of the method of developing various types of equilibrium in redistribution systems with real development in particular organisations or institutions enables to anticipate respective links, and identify and correct strategic behaviour with respect to their existence and parameters.

3 Examples of extension of the elementary redistribution system model

We now arrive at the most important. If our model of the elementary redistribution system is really suitable and useful abstraction, we may then extend it in various directions, so that it increasingly becomes the true picture of the reality. Each step of such extension then enables to imagine better what is going on in the real social systems. And in accordance with the aforementioned, each (methodologically efficient) extension of the elementary model will be the simplest (most economical). We select four examples from what we have been successful with so far. They differ by their level of achievement, or the degree of development, and all of them are instructive in a different way:

- The first example represents model extension using effects of the competition and inter-organisational migration. It is in a way the most developed and most elegant example. It is based on a very simple model extension that has however very strong interpretation. In addition, nice (also within the aesthetic meaning) mutual supplementation of the arithmetic and geometric expression of this extension takes place here.

- The second example represents model extension using the element of mutual corruption of players in forming alliances and distributing compensations. It is interesting, since the primary analysis of feedback option consideration (i.e. means gained by players in a single round to influence results in the next round) leads us to the opinion that the whole model becomes very complicated and that we will be forced to define and test a great number of cases to reveal the key issue (which enables reality "decoding"). Further examination however shows that minor extension of the model is sufficient again to capture very economically the most important matters in this extension that form players' behaviour.

- The third example pays attention to the possibility of players from one system type to influence forming alliances and compensation distribution in other systems. This example is in a stage when we are able to present basic tools of model extension and first results of their application. The model extension stated is not in a stage when the simplification effect takes place, i.e. when from the great volume of model specifications we select the option, which would be the key searched for, enabling to identify respective reality as economically as possible.

- The forth example points out the problem area of cross alliance effects (i.e. alliances of players from various redistribution systems) on the process of negotiation inside individual redistribution systems. This area of model extension is currently the less developed as compared to the other three. With respect to further development of the redistribution systems theory and its application, it is however the area of significant importance.

3.1 Example one: Competition and inter-organisational migration effects

In the article Budinský-Valenčík (2008), we have shown that in the systems not exposed to the pressure of competitive environment alliances of average and less performing players will be prevailing, while the most performing players will be discriminated. What would happen, if the system were exposed to the competition?

It becomes apparent that we must examine competition effects always in certain connections – e.g. with systems development and the possibility of inter-organisational migration (i.e. in connection with the fact that anyone may leave or join the social system). Inter-organisational migration has a decisive influence on acquisition and maintanance the human capital of companies and other types of organisations.

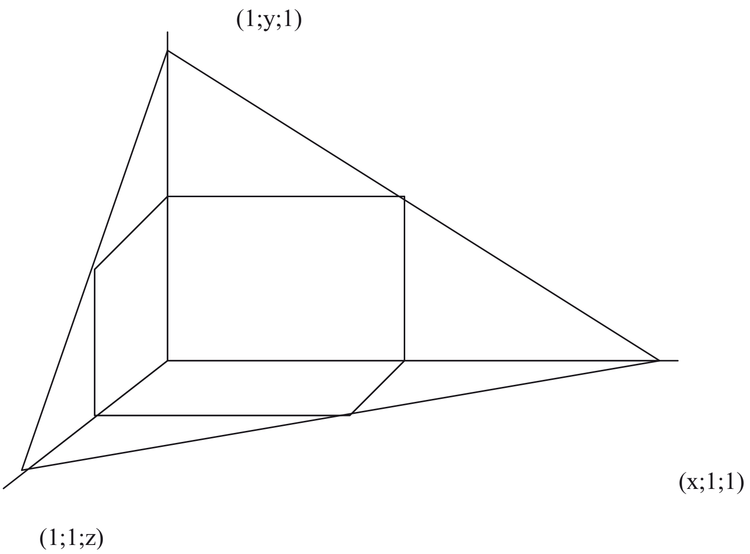
To create a respective model, it is important to find appropriate simplification that would serve as a key to precise description of the phenomenon. It became apparent that the most appropriate simplification is to put the possibility of inter-organisational migration in direct link with the lowest compensation, which must be paid to the most performing player. Players then negotiate how they can distribute the compensation available by forming various alliances. Such point of view is also nearest to the real situation. It can be expressed mathematically as follows:

If the most performing player had to receive compensation higher then 1, e.g. a, while the parameter $0 < a < xmax$ (the highest value that the player A may receive), we then get the following equations:

$$a + y + z = 12 - \eta.R(a; y - 4; z - 2)$$
$$x + 1 + z = 12 - \eta.R(x - 6; 3; z - 2)$$
$$x + y + 1 = 12 - \eta.R(x - 6; y - 4; 1)$$

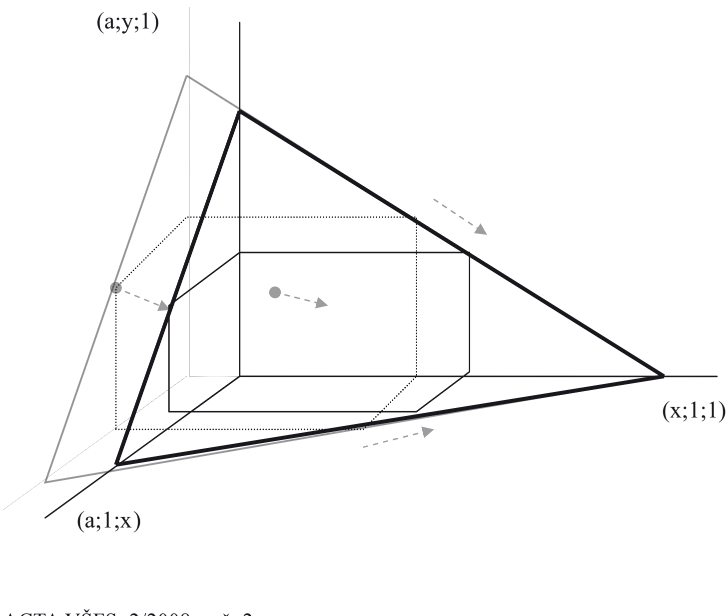
Very interesting and at the same time fully in accordance with what we have said on the importance of Diagram 1 is also the graphic expression of inter-organisational migration possibility. Let's show how we can get to it. The simplified model of equilibration in negotiation in pandering form can be displayed similarly as in Diagram 2 in the article Budinský- Valenčík (2008):

Diagram 2: Equilibration in negotiation in pandering form in the simplified model:



Instead of calculation, we may then use a visual image of where individual types of equilibration will move.

Diagram 3: Movement of discrimination equilibrations and Nash equilibration in the simplified model:



Thin dashed line indicates the original situation, the thick line indicates borders of the redistribution area moved at the parameter and with axes of coordinates (x;1;1), (a;y;1), and (a;1;z). It displays graphically the condition of discrimination equilibrations in negotiation with pandering, and arrows show movements of discrimination equilibrations and Nash equilibration.

The experience gained by extending the module in given direction can be summarised as follows:

- Extension base is a very minor modification of equations leading to discriminating equilibration discovery (single parameter value will change only).
- This equations change corresponds with a geometrical model that is very descriptive and intuitively understandable.
- We were able to obtain the result by searching for the answer how to express effects of the competition and inter-organisational migration as economically and simply as possible, or using the opposite method, i.e. to made certain formal modification of equations and try to interpret it.
- In our case, we were searching for model expression of what we have identified in the reality. The opposite procedure seems to be very tempting. The path from formal extension of the apparatus, which describes the reality mathematically, to interpretation of what this extension means is however not as simple as it might seem, if we know the result.
- In this case it consists in the fact that the model describes not only the competition or not only inter-organisational migration, but concurrence of both effects. This among others shows that good model must proceed from careful perception of the reality. For example, trying to determine competition or inter-organisational migration effects, etc. in "clear" form at any price would not lead to any success in this case. It is always interaction between the model extending by "clear" mathematic generalisation and the effort to use the model to capture the essentials that we identify in the reality itself.

3.2 Example two: Possibility of mutual corruption of players (investing into a position)

The real voting power or players' influence on game results may usually vary. Even in the case where all players have formally the same (equal) position. Various methods are for example used for promoting effects of assets, income level or disposable funds on the possibility to influence forming alliances and distribution of compensations. In the event that the income may influence game results (which happens in practice almost any time), the feedback actually functions between the result of the previous game and the parameters of the next game. We know that feedback existence has always a significant impact on the behaviour of any system. We may anticipate that this will also apply to redistribution systems. The point is to find an appropriate model that would describe this phenomenon; i.e. that would capture consequences of investing into the player's position in that system.

The solution is always simple, and it also applies to this case. We may understand income effects on game results (distribution of compensation) in the next round (in forming alliances and achieving certain compensation within the alliance) so that the player will use part of his/her funds, which he/she disposes of and which he/she gained in the previous round, to pay to another player for the fact that this other player applies the strategy required by the first player. A very simple principle. It is however not simple to put it in operation, also with respect to necessary assumption of simplicity.

One of the problems we have to cope with is for example the question – how to use the model to capture the problem of agreement compliance or violation, i.e. that the player will apply the strategy required in return for compensation.

The following rules have been proposed:

1. An initial state was determined (e.g. distribution of compensation in the ratio 3:2:1).
2. Anybody may try to bribe anyone else from the funds available to him/her:
2.1. Directly from the amount he/she actually disposes of.
2.2. And also from the amount gained from negotiation.
2.3. By promising compensation after round completion.
3. Any agreement related to compensation distribution may be proposed and any agreement of this type may be violated at any time.
4. The player not violating the agreement is expected not to violate it, until he/she violates some agreement; the player violating the agreement is expected to violate agreements.
5. After completion of each round, every player will receive his/her compensation
6. Each player spends certain amount in each round (e.g. 1).
7. All players are informed of all steps (negotiation results, agreements conclusion, their violation...) of other players.
8. Each player strives:
8.1. Not to get into the state of exigency.
8.2. To maximise his/her compensation in a long-term horizon.
8.3. To maximise the total of compensations in individual rounds.

Let's now show an example of a game that was played in public at the regular Theoretical seminar of PCE in VŠFS on 28.01.2008:4

4 *Theoretical seminar of PCE (Productive Consumption Economy) is held every week from October 2003 during teaching. Information on this event (topics, details) can be found at www.vsfs.cz → Science and Research → Theoretical seminar of PCE.*

A	**B**	**C**	
3	**2**	**1**	**(initial state)**
3(-2)	2	1(+2)	A offered 2 to C, B did not respond, C accepted
1(+3)	2(+1)	3(+2)	compensations in this round
4(-1)	3(-1)	5(-1)	consumption
3	**2**	**4**	**initial stat in the next round**
C offers 1 to B			
A offers 1 to C			
B did not accept, requires 2 from C			
C did not accept, offers 1 to A			
A is willing to accept			
B did not respond			
3(+1)	2	4(-1)	
4(+2)	2(+1)	3(+3)	compensations in this round
6(-1)	3(-1)	6(-1)	consumption
5	**2**	**5**	**initial stat in the next round**
			A offers 2 from the result to C
			B offers 1 to C
			C accepts the offer of 2 from the result from A
5(+3)	2(+1)	5(+2)	compensations in this round
8(-1)	3(-1)	7(-1)	consumption
7	2	6	initial stat in the next round
7(-2)	**2**	**6(+2)**	**A complied with the agreement**
5	2	8	

In this game, the necessity to particularize determination of the negotiation process has proven among others.

If we consider the influence of redistribution to reduced performance of the organisation and the possibility to negotiate with pandering, we will find that the system would behave differently. We have to however assume that all players are perfectly reasonable, perfectly informed of the results of mutual negotiations, and the negotiation process is not limited whatsoever. Using the following rationale, we can show you how the system will behave under different circumstances. We will search for respective equilibriums, their definitions, interpretation and calculation. We logically also assume that some discrimination equilibrium was formed within the system, e.g. based on the alliance B and C against the player A. (Should another alliance be formed, the deliberation method and the results achieved will not change.)

Case one: Players B and C will tend to pander by bribing the player A, and they try to gain higher compensation then between each other within the existing alliance. Forming such alliance would also mean an improvement for the player A. He/she has no reason to decline the offer, which will bring him/her the compensation higher then 1 from any of the players B or C in the next round. But how much can he/she ask for? Exactly as much as defined by calculation of the discrimination equilibrium in case that he/she is in the winning alliance. The system thus turns into the state of any discrimination equilibrium.

Case two: Players B and C may offer a certain amount to the player A from their own funds before getting compensation in the next round, to assure getting into the alliance with him/her and that the player A will improve his/her position in this alliance. The player B has seemingly an advantage, since he/she disposes of greater funds and can assure the result by "overpaying" the player A. But also in this case there is noting new. The thing is that all players think reasonably and are perfectly informed of mutual negotiations. The player B must aim to improve his/her compensation as compared with the previous round. He/she may overpay the player A, giving him/her more then the player C who would have to pander. In such case, his/her compensation would however be lower then if he continues with the alliance with the player C, and thus such situation will not arise. In the event that we do not consider the difference of values in time (i.e. the amount obtained in advance from the compensation in the first round or from the compensation in the second round with the same nominal value has also the same real value), the total of compensation in advance and after must be exactly at the discrimination equilibrium level.

Example three: The player A who is discriminated will try to improve his/her position and declares willingness "to get bribed" with certain amount, e.g. 2. In such case, he/she would be in the winning alliance with higher compensation, together with the player who would first accept his/her offer. If however some negotiation with pandering takes place and other two players could give him/her more, discrimination equilibrium would be formed again with the same parameters as when we do not consider bribery.

Example four: The case when a player outside the alliance (in our case the player A) makes an offer to get bribed with certain amount and must follow this offer is even more interesting. I.e. even in the event that another player (B or C) offers more, he/she cannot withdraw from the agreement declared. Then the player who first makes an offer to this player will form an alliance with him/her. However, we have to think even further here as well:

- It subsequently raises a question – how much he/she may ask for. It is interesting that this amount reaches up to the discrimination equilibrium level. I.e. as soon as he/she offers any amount that is a bit lower, one of the players will form equilibrium with him/her.
- Also other players may be aware of the above and make respective offer preventively. In such case, the system will "fall" again within some discrimination equilibrium.

Example five: Agreements however do not have to be complied with. In the first approach, we may model this situation by the fact that the amount, which one of the players is bribed with, must be higher exactly by much the risk of non-compliance with the agreement is higher. But if the game is played repeatedly, its development will be interesting again. The player who would not comply with the agreement might have never got into the winning alliance. So we can see that there is a very strong spontaneous pressure to comply with agreements in the system. To be more specific – agreements will be complied with in a purely model situation (which itself is an important and not trivial result).

Model investing into the position or the feedback bring interesting results also with respect to behaviour of real systems, and they may be summarised as follows:

1. Discrimination equilibriums influence very strongly the nature of agreements and subsequent distribution of compensations even with many modifications in the course of negotiation.
2. Discrimination equilibriums may be modified by some exogenous effects such as credibility of players, difference of the real and nominal value in various rounds, etc. These effects can be included in the system very easily, and they shift discrimination equilibrium values.
3. The system is very sensitive to exogenous effects that favour one of the alliances, e.g. to network links (an alliance with players from various redistribution systems).

The experience acquired by model extension in given direction can be summarised as follows:

- Even a very simplified model of bribery, or investing into the player's position, has many modifications, and the task is to compare it with the reality to find those that are relevant.
- Although it seems in the first approach that model extension using the element of possibility of mutual bribery of players would result in large number of variants of their behaviour, we can find in the detail analysis that even in such extended model, the same discrimination equilibriums and the same Nash equilibrium remain as in the elementary model. Some parameters may be only be slightly modified.
- It shows at the same time that what was considered sufficiently accurately expressed and obvious (in this case intuitive vision of the negotiation process) it requires determination in much more details.[5]

3.3 Example three: Influence of players from one redistribution system on forming alliances in another system

We have so far considered cases when in certain system the players of that system have sovereign influence only. We however know that the reality is different. There is usually certain influence of players from other systems on the behaviour of players in the system in question. It is particularly due to "networking" of players from various systems, i.e. by forming alliances "across" various systems, the members of which are players from various systems. These alliances may be visible or hidden. The may however significantly influence development in any particular system. We can model this situation using for example the following method:

We have N players in a redistribution system (A_1, A_2,…A_N), while the influence power of each of them is:
$1 + a_i q_i$,

where:
a_i is a coefficient of the level of influence of i-th player of the cross-alliance (in the simplest model) on forming alliances and the amount of compensation in his/her redistribution system;

5 *For negotiation problems see for example Horniaček (2006), Osborne (2002), Selten (1999a), Selten (1999b).*

q_i is the number of players from other redistribution systems, which this player forms the cross-alliance with.

(The above shows nothing but the fact that the more players from various redistribution systems form the network, the higher the power of their influence inside each redistribution system.)

The condition for forming a minimum winning alliance can then be formulated as follows:

$$\sum_j (1 + a_{ij}q_{ij}) > 1/2 \sum_i (1 + a_iq_i)$$

while the following applies for all k:

$$\sum_j [(1 + a_{ij}q_{ij}) - (1 + a_{ik}q_{ik})] \leq 1/2 (\sum_i 1 + a_iq_i)$$

where:
the index i assumes the value 1, 2,…N;
the index j gradually assumes the value 1, 2,…N_j ($N_j < N$) and indicates only those players from that redistribution system who are members of the minimum winning alliance;
the index k assumes some values 1, 2,…N_j ($N_j < N$) and indicates only some (any) player from that redistribution system, who is a member of the minimum winning alliance.

Both conditions are quite apparent. The first one (that the alliance is winning, i.e. disposes of sufficient influence) says that total influence of all players forming the minimum winning alliance must be higher then a half of all players in that alliance. The second one (that the alliance is minimal) says that if any member leaves this alliance, it will not have sufficient influence anymore.

If $a_i = 0$ or $q_i = 0$, we have the original model. Either there is no influence of the cross-alliance on development inside the system, or no cross-alliances have been formed. In compliance with the above conditions, we can modify a system of equations leading to calculation of discrimination equilibriums.

The above system extension distinguishes itself also by significant "economy". We assume that forming a cross-alliance of certain player from the distribution system in question with each player from other redistribution system increases its influence the same way, or by the same value (a_i). If $a_1 = a_2 = … = a_n = a$, it means that this influence increase would be identical for each player from that redistribution system. In that case, it would be possible to write down the conditions of the minimum winning alliance using even a simpler method.

Based on the above presented formalism, we can formulate many interesting tasks. It is apparent that reasonably behaving player will try to maximise the number of players he/she forms the cross-alliance with. He/she may also form an alliance with more players from the other redistribution system (exclusivity does not have to be preserved). The same applies reciprocally. What would be the consequences, if there were players from different alliances in such developed cross-alliances (expulsion from the winning alliance as well as members of the winning alliance)? How will this influence stability of winning alliances? What role would this play in negotiation? What consequences will player's awareness or lack of information on the existence and structure of cross-alliances have (i.e. differences due to existence of hidden and visible alliances)? To answer these and similar questions, it is necessary to formulate other supplementary specifications describing players' behaviour. These specifications (formalised rules) must bet intuitively understandable, simple, and correspond with the reality.

The experience acquired by model extension in given direction can be summarised as follows:

- As soon as we open the door to model extension by suitable formalisation, there is usually a great number of options of how to continue. It was also similar in the previous second example.
- Various options of paths and model specifications must be examined step by step.
- In doing so, there are questions that would not arise without model extension. These questions are usually both, interesting and significant, with respect to practical interpretations.
- Sooner or later we can assume that a method will appear how to describe this problem area and express it by relatively simple way, when many considered modification options of extended model description will prove to be misleading.

3.4 Example four: Influence of links between redistribution systems on the negotiation process inside each of them

One of the basic assumptions for calculation of discrimination equilibriums in the elementary redistribution system is that the negotiation process will pass perfectly, without any limitations. The reality is usually different. Not all players are informed of the negotiation process, the negotiation requires certain transaction cost, players do not have unlimited time, they might be more or less sympathetic to each other for various reasons, to trust each other differently, etc.

With respect to the fact that all three equilibriums are equally probable under the conditions of perfect negotiation, imperfections in the negotiating process as well as various external effects are decisive in what equilibrium would be formed. Even very minor effects. It is highly probable that network links (cross-alliance) of players from different redistribution systems may influence forming alliances also by the fact that they affect players' preferences in negotiation. If we could find suitable model expression, it would be possible to answer the following questions:

- How do redistribution systems link together within the meaning that forming certain type of alliance in one type of systems transfers to formation of the similar type of alliance in surrounding redistribution systems?

- What external influences on the redistribution system do move discrimination equilibriums and Nash equilibrium only (they can be compensated by the fact that any player would ask for lower compensation to enable influence of the winning alliance with his/her participation), and what external influences do eliminate formation of some types of winning alliances (and thus cannot be compensated)?

Both questions are from the theoretical and mainly practical point of view very important. In this sense, the above can be formulated also in reverse order: One of the important aims of development of the redistribution systems theory is to express the negotiation process as respective model extension, so that it opens the path to answer the above questions. If we attempt to answer the second question (what external effects can be compensated and what cannot), we may consider for example the following: Effects that cannot be compensated by concessions in negotiation of any player may relate to the fact that by creating network links between redistribution systems the respective player is excluded from the negotiation process. Another option is that he/she has no information on how the negotiation process goes and whether there is any at all. There are probably also other reasons for what predetermines certain alliance types.

The experience gained by extending the module in given direction can be summarised as follows:

- The option to ask the questions above is subject to achieving certain level of model development of the redistribution systems theory. As it is usual with any theory development, asking questions is important, it is certain theoretical result, and it contributes to better understanding of what is going on in the reality itself.
- Verbal (not formalised yet) description contributes to better classification of the problem area, but its informative ability is limited.
- For better capturing of the problem area, formalisation and subsequent mathematic expression prove not only desirable but also necessary.

4 What we have achieved

There are various concepts of how the theory develops. Each expresses the reality with certain simplification and complies more or less with that particular case. Although none of them is perfect, conclusions and suggestions result from each good concept, concerning the process in any theory development, or an answer to the question whether the theory in that particular case is prospective or not.

In our contribution, we have presented certain view of development of such theory types that use an abstract, formalised, and subsequently also mathematically expressible model for reality decoding. Consecutively, we showed how the elementary model is extended in case of the redistribution systems theory, how it gradually opens the path for answering the questions formulated, but also for asking other questions, and what role the visualisation and the possibility to confront the model with the reality play in this process.

For this, we have used four concrete examples documenting and illustrating the process of model extension. Each of them corresponds with different state of knowledge of such part of the reality that we describe by extending the model, using formalisation and mathematic expression.

We assume that this is a suitable form of getting the expert public acquainted with continuously achieved results, since in the stage of dynamic development of the theory, each progress within the meaning of the answer to certain question raises even more questions. Therefore we cannot wait until we know answers to all of them. With respect to the fact that it is in our opinion a very prospective area of recognition, methodological reflection and process documentation play also a very significant role. Last but not least, using this form we aim to interest, motivate, and inspire those who are interested in co-operation in the area of redistribution systems theory. There is namely the possibility to achieve demonstrable and original theoretical results.[6]

Abstract
The contribution resumes the article called Nash Equilibrium in Redistribution Systems (Calculation, significance, use), published in the magazine ACTA 1/2008. It is based on some general methodological questions of theory development that use mathematical apparatus, where finding an appropriate reality simplification plays an important role. The article then applies these bases to the problem area of redistribution systems theory, which is one of the game variants. It specifies more the expression of discrimination equilibriums and Nash equilibrium, presents the original scheme that enables visualisation of these equilibrium types, extends the model of the elementary redistribution system by considering some influences on the redistribution system. Specifically, these are competition and inter-organisational migration effects; possibility of mutual bribery of players (investing into the position); influence of players from one redistribution system on forming alliances in another system; influence of links between redistribution systems on the negotiation process inside each of them.

Keywords
Theory of games, redistribution systems theory, Pareto optimum, Nash equilibrium, competition, inter-organisational migration, coalition, negotiation

JEL Classification / JEL klasifikace
D01, D33, D74.

Souhrn
Příspěvek navazuje na článek Nashova rovnováha v redistribučních systémech (Výpočet, význam, využití) uveřejněný v časopisu ACTA 1/2008. Vychází z některých obecných metodologických otázek vývoje teorií, které využívají matematický aparát a v nichž důležitou roli hraje nalezení vhodného zjednodušení reality. Tato východiska pak konkretizuje na problematiku rozpracování teorie redistribučních systémů, která je jednou z variant teorie her. Konkretizuje vyjádření diskriminační a Nashovy rovnováhy, prezentuje původní schéma, které umožňuje názornou představu o těchto typech rovnováhy, rozšiřuje model elementárního redistribučního systému o uvážení některých vlivů působících na redistribuční systém. Konkrétně pak jde o vliv

6 The work flow can be monitored at www.vsfs.cz → Science and Research → Theoretical seminar of PCE → An archive, where up-to-date research results are published in annexes to individual lectures.

konkurence a meziorganizační migrace; možnost vzájemného uplácení hráčů (investování do pozice); vliv hráčů z jednoho redistribučního systému na vytváření koalic ve druhém systému; vliv vztahu mezi redistribučními systémy na proces vyjednávání uvnitř každého z nich.

Klíčová slova

Teorie her, teorie redistribučních systémů, paretovské optimum, Nashova rovnováha, konkurence, meziorganizační migrace, koalice, vyjednávání

Contact address / Kontaktní adresa

h. doc. RNDr. Petr Budinský, CSc.
Institute of Finance and Administration, o.p.s., Vice-Rector for Education Affairs and External Relations (e-mail: petr.budinsky@vsfs.cz)

doc. Radim Valenčík, CSc.
Institute of Finance and Administration, o.p.s., (e-mail: radim.valencik@vsfs.cz)

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