Redistribution Systems and Contextual Games

Redistribuční systémy a kontextuální hry

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This article was supported by the Czech Science Foundation (Theory of Redistribution Systems, No. 402/09/0086).

Abstract

The goal of this article is to demonstrate how mutually inter-related the two directions taken by the expansion and perfection of the game theory apparatus are, on one hand in the examination of redistribution systems and on the other hand in the analysis of contextual games. And to take advantage of this opportunity to present the latest findings attained in both areas and at the same time to characterize one of the most important types of the objects that our team has come across in the execution of the program as defined above – structures based on the mutual covering-up of the breaching of rules and generally accepted principles. These are objects without the theoretical grasp of which it is not possible to adequately describe the essence and social contexts of corruption and similar phenomena. And therein lies the social relevance and the originality of the topic. From the perspective of the development of science, the mentioned approach makes it possible, among other things, to explain some apparent discrepancies between game theory and this theory inspired by experiments.

Keywords

redistribution systems, contextual games, parallel games, structures based on mutual covering-up

Abstrakt

Cílem předloženého článku je ukázat, jak spolu vzájemně souvisejí dva směry, kterými se ubírá rozšíření a zdokonalení aparátu teorie her, a to na jedné straně při zkoumání redistribučních systémů, na straně druhé pak při analýze kontextuálních her. Při této příležitosti pak prezentovat nejnovější poznatky dosažené v obou oblastech a současně charakterizovat jeden z nejvýznamnějších typů objektů, na který náš tým při realizaci výše tematizovaného programu narazil – struktur založených na vzájemném krytí porušování pravidel a obecně přijatých zásad. Jedná se o objekty, bez jejichž teoretického uchopení nelze adekvátně popsat podstatu a společenské souvislosti korupce a obdobných jevů. V tom spočívá společenská aktuálnost i původnost tématu. Z hlediska rozvoje vědy pak uvedený přístup umožňuje kromě jiného vysvětlit některé zdánlivé rozpory mezi teorií her a touto teorií inspirovanými experimenty.

Klíčová slova

Redistribuční systém, kontextuální hry, paralelní hry, struktury založené na vzájemném krytí

Introduction

In 2006 the team at VSFS started to execute the program that could be briefly characterized as the deciphering of human behavior using game theory, i.e., clarifying why people and groups of people behave the way they do. Neither the problem nor the idea to use game theory for its resolution are original. Explaining human behavior is an ancient topic of most of the social sciences and game theory has also posed – and continues to pose – this question ever since it was formed as a scientific discipline. The original in the approach of our team was and is that:

First: It establishes the said topic comprehensively within the context of the issue of the preparation and implementation of reforms in the area of social investing and social security.
Second: For its resolution it develops its own modifications or expansions of the game theory apparatus so that it is possible to grasp the given issue using exact mathematical instruments.

The article is related to the series of four articles in the ACTA periodical, in which theoretical sources of the approach based on examination of redistribution systems by means of game theory are described in detail. The article is partly based on certain latest contributions (Valenčík-Budinský 2010, Heissler-Valenčík 2010, Šnajdar-Valenčík 2011), which were published recently or which were being published together with it. Some of the pictures from them have been adopted (as indicated for each picture), a part of them is presented for the first time (as also indicated). The texts are original and contain findings published for the first time, only a minor part has been adopted, revised and completed based on previous publications.

1 Briefly on the Context in Which the Program of the Deciphering of Human Behavior Is Being Executed

A major part of the reforms, the significance of which is currently being emphasized, are changes in the area of the pension system, the system of financing of health care and the system of financing of education. Reforms in these areas are usually conceived independently and the urgency and significance of the reforms are usually discussed according to the amount of money that is being redistributed via corresponding systems.

The need to apply the game theory apparatus for the analysis of human behavior emerged during the work on two successive projects of the CR GA: Effectiveness of Investing into Human Capital (carried out in 2003-2005) and Investing into Social Capital and Effectiveness (carried out in 2006-2008). One of the results of the work on these projects was the proving of the existence of significant linkages among the reform of the pension system, reform of the financing of health care and the reform of the financing of education, as these are systems of social investing and social security supported by public resources. In these systems one of the important criteria of the effectiveness of social investing (in the area of education as well as health care) is to what extent they contribute to the stability (sustainability and equilibrium) of social security systems (i.e., the financing of health care itself, as well as of the pension system). From this perspective arises the fundamental

importance of the issue of the financing of investments into education within the context of reforms in other areas. From a practical point of view, this means, for example, creating a motivating environment for the synergetic functioning of the aforementioned systems of social investing and social security so that they support the possibility of extending the horizon (possibly even the zenith) of the voluntary productive engagement of people. Practically focused projects were also carried out based on the aforementioned generally theoretical results.

Experience from discussions during the preparation of reforms as well as with what occurs during their practical implementation shows that by far not everyone who is capable of influencing the given area is concerned with the most effective possible resolution of problems. This concerns not only the provable politicization of the reforms issue, but also – and this is also provable – the building-in of various mistakes into various reform activities that should make it possible or actually do make it possible to extract means from the reforms implementation in favor of various lobbies. And these are not random and unconscious mistakes, but rather intentional and systematic mistakes, frequently thinking several steps ahead and counting on various alternatives of development.

These findings brought our team – which was put together during the work on the above mentioned GA CR projects – to the conclusion that if the given issue is to be dealt with theoretically as well as practically, the game theory apparatus needs to be applied to it. As far as theoretical research is concerned, this means not only answering the question of "how should it be", but also the question of "who is preventing the implementation of an effective solution and how and why are they doing so and who is able to implement an effective solution in practice and how". As far as the practical aspect is concerned, this means identifying – with a help of the game theory apparatus – who the real players are in the given area, what their objectives are, what strategies they are using, what the cause of the conflicts that take place here is, etc.

Since 2009 and within the above stated context and with the above stated objectives, work is being carried out at VSFS on the GA CR Redistribution Systems Theory project. This is an original direction of expansion of the game theory apparatus that sets as its goal to comprehend what is taking place in reality from the perspective of the aforementioned issues.

2 Possibilities and Limits of the Redistribution Systems Theory

The Delimiting of Redistribution Systems (Valenčík 2008) was based on the need to create a model that would make it possible – within the context of the issues mentioned in the previous part – to identify, describe and analyze what is taking place in reality. The following were selected as the basic elements:

1. The possibility of creating coalitions that give preferential treatment to their own members at the expense of other members.
2. Expressing the process of negotiations during which coalitions are formed and the distribution of payouts occurs.

3. Existence of the dilemma between a player's own profit and the effective functioning of the entire system.

In other words, a redistribution system is a system in which negotiations about the formation of coalitions and the distribution of payouts occurs and some players may receive preferential treatment and others may receive unfavorable treatment, i.e., be discriminated against. The greater this preferential treatment and discrimination is, the more the system performance will decline. We model this using a corresponding redistribution equation, which determines the redistribution area.

Selection of the basic elements is subject to two conflicting criteria. On one hand, there should not be too many of them, because in the opposite case it would not be easy and probably even realistic to create an initial consistent model of the corresponding system (initial in the sense that with its gradual expansion we are able to also model systems with more elements). On the other hand, it must ensure the sufficient explanatory power of the mathematical model, so that the results can be interpreted from the perspective of the practical context suggested above. In other words, what we input into the model should not make it too simple or too complex from the perspective of the use of mathematical means in the analysis of social reality.

For the three player case and some other restrictions (full informedness of players, their equal voting, respectively influential, powers, non-existence of time delays or transaction costs) we were able to create several types of functional models and prove some important assertions. The most significant results we attained can be briefly characterized as follows:

1. If the players are forming fully discriminating coalitions and if the negotiations of the players are governed by rules that are acceptable from an intuitive perspective, the sequence of the payouts of players based on the coalitions negotiated between any two players converges to three points on the redistribution area, and the set of these points is equal to the sole and final internally and externally stable set. These three points were named discriminating equilibriums.
   Several comments to this:
   - A fully discriminating coalition of two players in a system with three players is one where the player outside of the coalition receives the smallest possible payout.
   - When speaking of intuitively acceptable rules, we mean the following: Each player a) proposes such a distribution of payouts where he and one of the other players becomes better off; b) presents the proposal to the player with which he can be the better off player; c) the proposal of his payout ranges between the biggest one that he could get under these conditions and the biggest one that he could get if he negotiated with the third player (with the player with which he could get the smaller largest possible payout).
   - The set is internally stable if it is not possible to say about any of its elements that it is better than any other element of this set. The set is externally stable if it holds for every element of the original set (in our case the redistribution area) that among the elements of the externally stable set there is at least one that is

better. An element of a set is better than another element of a set if at least two players have a bigger payout in it.

2. The players can also negotiate a jointly acceptable equilibrium (our term), in which they will be better off in comparison with the average payout if they attempt to proceed as stated in the previous point. The jointly acceptable equilibrium point is determined unambiguously.
   Several comments to this:
   - The average payout is determined by the sum of his (two) payouts in the case where he is a member of the winning coalition, and the payout where he himself is fully discriminated against
   - There is a precisely defined and intuitively acceptable strategy via which the players arrive at the jointly acceptable equilibrium, and this strategy is clearly determined by these negotiations (Valenčík-Budinský 2010). Interesting and important is the fact that it is also possible to arrive at this equilibrium using several other approaches.
   - This equilibrium has certain specifics in comparison with the Nash solution, the Kalai-Smorodinsky solution, the Shapley value.
3. The process of negotiating fully discriminating coalitions as well as the jointly acceptable equilibrium is very sensitive to external influences. One of the most important objectives of the theory is to reveal and describe which influences are involved and what the mechanism of their functioning is, particularly the mechanism of the functioning of those influences that cannot be compensated by concessions during the negotiations.

A comment to this: differentiating the influences that can be compensated by concessions during negotiations and those that cannot be compensated is absolutely fundamental. And the answer to this question has an interesting interpretation, to which we will get shortly.

4. The functioning of external influences can be – at least in the first approximation – described using parallel games, i.e., games during which some of the players get payouts from the system without the other players being aware of this.
   Several comments to this:
   - A redistribution system containing parallel games can be described, for example, by the following set of equations:

$$\sum_{j=1}^{M} x_{0j} = \sum_{j=1}^{M} e_{0j} - \eta_0 R_0 \left(X_0 - E_0 \right) - \sum_{i=1}^{N} \pi_i \sum_{j=1}^{M} x_{ij}$$

   An expanded set of redistribution equations for the case of parallel redistribution games can be read as follows: in the basic redistribution game the sum of the payouts of players ($\sum^{M} x_{ij}$) equals how much they could get if they received payouts according to their performance ($\sum_{j=1}^{M} e_{0j}$), but decreased as a result of the deviation of the payout of players from the performance of players ($\eta_0 R_0 \left(X_0 - E_0 \right)$) and also

by the negative influence of parallel redistribution games on the total performance of the basic redistribution system ($\sum_{i=1}^{N} \pi \sum_{j=1}^{M} x_{ij}$).

- The set of the equations listed above describes only the "morphology" of that which plays a triple role: it predetermines visible coalitions in redistribution systems, it diverts means from the system in favor of some players and their groups and it decreases the system performance.
- An important phenomenon – although in retrospect not all that surprising – is the fact that the element of the differing informedness of players enters the game. For fundamental reasons, only some players can know about that which predetermines the formation of coalitions. At this point, however, we do not have a sufficient idea about the form of the structures that are created this way. There is nothing left to do but to start searching for them.

Already at the beginning of the elaboration of the redistribution systems theory it was clear that it will have to be expanded by additional elements. And not only in the sense of adding more partial aspects to it, but also in the sense of fundamental generalizations. The question of "What should be added to the redistribution systems theory?" – as an unexpressed but frequently also expressed question – accompanied practically all discussions within our team or presentations of results to the public. It gradually crystallized into an interesting position:

- Partial expansion of redistribution systems theory is connected with the inclusion of those influences and elements that are "visible", i.e., about which all players are informed.
- The fundamental expansion or generalization of the approach is then based on the revealing of what is hidden and even tries to remain hidden.

In other words: the reality with which we are dealing can be viewed from the perspective of a certain polarity between what can be seen and what cannot be seen, between what all players are informed about and what some players are not informed about. The redistribution systems theory is then proving to be one of the poles of this polarity, as well as a suitable "reading prism" capable of revealing what remains hidden and keeps hiding. The discovery of this fact is one of the most significant results of redistribution systems theory, as well as a self-reflection of its limits.

3 Contextual Games and Identification of Structures Based on Mutual Covering-up

The issue that lead to the discovery of the contextual character of some games was faced by our team already when dealing with the apparent discrepancy between theory and experiments, which is described in the Valenčík-Budinský (2010) article. The decisive step – based on T. Kosička's (2010) initiative – was then taken in connection with the problem of the interpretation of experiments based on the prisoner's dilemma motives.

What is the issue? In experiments examining the behavior of people in situations of the prisoner's dilemma game type, we came across a considerable discrepancy between what theory says and how people actually behave. In the prisoner's dilemma, both players have two options – cooperate or betray. The selfish choice to betray leads to greater benefits than cooperation if the second player cooperates, but to lesser benefits if the second player also betrays. The rational behavior of both accused persons is to testify against the other person, even though the optimal solution for both of them together is for both to remain silent. The result where betrayal is the right decision has lead to many discussions and explanation attempts. Several widely published experiments were also performed. Let us show the results of the most well-known and most important experiments, as per the following table.

Table 1: Results of experiments on the prisoner's dilemma motives

	non-cooperation	cooperation	unknown decisions
Shafir, Tversky (1992)	97	84	63
Li, Taplan (2002)	83	66	60
Busemeyer (2006)	91	84	66

Source: Kosička (2010).

The names and year show who performed the corresponding experiments and when. The numbers in the columns express the percentage representation of "betrayals", i.e., cases when the corresponding player who had guaranteed information that the other player betrayed him (first column) or did not betray him (second column), or was not informed about the second player's decision (third column), opted for the strategy of non-cooperation. Other experiments showed that the willingness to betray or cooperate is influenced to a large degree by the size of the reward (punishment).

Let us now take a closer look at where is the difference between how players (i.e., specific people) should behave in theory versus how they actually behave:

1. If we do not know how the other player decided, we should always betray (and not only in 60-66 % of cases of non-cooperative behavior).
2. If we do know that the other player betrayed us, we should betray all the more so (and not only in 83-97 % of cases of non-cooperative behavior).
3. If we know that the other player is cooperating, why betray him (and why in 66-84 % of cases of non-cooperative behavior, i.e., in even in a greater number of cases than when we don't know how the other player behaved)? (For the sake of precision, let us add that this concerns not only a discrepancy between theory and experiment, but – at least at first glance – a glaring discrepancy between two cases tested experimentally.)

How can the "irrational" behavior be explained (if, however, it really is irrational behavior)? We have to proceed from the contextual character of games, because in reality a situation where a prisoner's dilemma type game is played without repetition and is completely isolated from other games occurs very rarely. In most cases during the course of a game, other people (who we can consider to be players in other games) are observing how indi-

vidual players decide, and based on this they also create a relationship to the participants of the given game. In reality, it is not a clean game without repetitions played by certain two players that is taking place, but rather a number of games in which other players also participate and these games also include game situations that could be called contextual games with quasi-repetitions.

Therefore we can view each game that we are playing in reality as a contextual game, i.e., a game that we are playing in the context of other games. We are introducing the term "contextual games" as an original term. In theoretical literature we will find only the designation of some starting points with which we are working, such as Meliers – Birnabou (1981). But this is only a partial view without an apparatus that would make it possible to analyze the contextual games phenomenon.

The reflection itself of contextual games depends considerably on our experience and the transformation of this experience into the "on-line" mechanisms of our (human) decision-making in which important roles are played by imagination, emotions and other attributes of the psyche. Let us demonstrate how a game of the prisoner's dilemma type changes if we begin considering it as a game played in context with some other games.

Table 2: Payout matrix of a game of the prisoner's dilemma type on the example of the keeping or breaching of an agreement

		X_2	
		cooperation	non-cooperation
X_1	cooperation	6; 6	0; 8
	non-cooperation	8; 0	3; 3

Source: Created by the author.

X_1, X_2 are players that have two strategies – comply with an agreement or breach an agreement (breach agreed upon or acknowledged rules). Their payouts are presented in the matrix. Now let us assume that from the perspective of one of the players (e.g., X_1) the game has a certain context, respectively is played as a contextual game in the sense that the community in which this player lives may be (but also does not have to be) informed about the game's outcome. If he complies with the agreement and the other players in the given community see this, it will contribute to the increasing of his credibility capital (reputation). If, conversely, he does not comply with the agreement and the other players in the community find out about this, his credibility capital (reputation) will decrease. Let us also assume that the credibility capital (reputation) can (at least approximately) be valued in units in which the payouts from the prisoner's dilemma type games are made, and the corresponding player also values it. For example, the player values the loss in the event of non-compliance with an agreement with -6 points, and the increase in the event of compliance with the agreement with +2 points (trust is lost faster than it is gained). The following table shows how the situation changes.

Table 3: Payout matrix of a game of the prisoner's dilemma type with the taking into account of credibility capital (reputation)

		X_2	
		cooperation	non-cooperation
X_1	cooperation	6+2; 6+2	0+2; 8-6
	non-cooperation	8-6; 0+2	3-6; 3-6

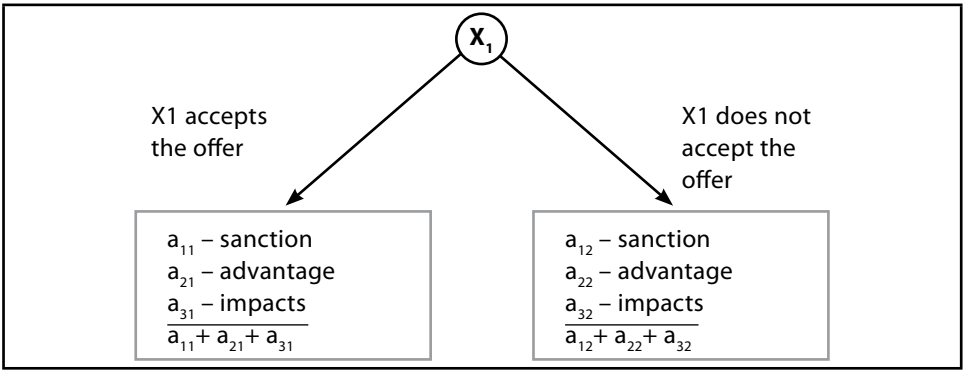
Source: Created by the author.

We can see that the situation changes dramatically. It is worthwhile for both players to cooperate. But only if the original payouts and payouts connected with the gain or loss of credibility capital (reputation) have certain values. The situation can be different if the values are different. As tends to be the case, the functioning of credibility capital (reputation) is dependent on certain assumptions. These include particularly the following:

- The possibility that one of the players (e.g., X_2) discovers that another player (e.g., X_1) is breaching agreements or rules.
- The possibility of the spreading of information by a player (e.g., X_2) about the breaching of agreements by another player (e.g., X_1).
- The possibility of sanctions issued by other players (community in which contextual games are taking place) against the player breaching agreements (e.g., X_1).

But the player who discovers the breaching of rules (let us call him player X_2) also has a possibility other than spreading information about the breaching of rules. Instead of spreading information about the breaching of rules, he can persuade the player that breached the rules (let us call him player X_1) to take action that is advantageous for player X_2. In the next illustration no. 1 we will demonstrate the dilemma of a player who is deciding whether to allow himself to be blackmailed or not.

Figure 1: Schema describing the blackmailed player's dilemma



Source: Created by the author

Here:

a_{11} is the valuation of the sanction that the blackmailed player will face if he accepts the blackmailing player's offer (in the given case one can assume that it will be zero)

a_{12} is the valuation of the sanction that the blackmailed player will face if he does not accept the blackmailing player's offer (in the given case one can assume that it will be quite severe)

a_{21} is the valuation of the advantage that the blackmailed player will have if he accepts the blackmailing player's offer (in the given case one can assume that it will have a certain value)

a_{22} is the valuation of the advantage that the blackmailed player will have if he does not accept the blackmailing player's offer (in the given case one can assume that it will be zero)

a_{31} is the valuation of the risks and negative consequences that the blackmailed player assigns to the fact that he allows himself to be blackmailed and starts to cooperate (in the given case this concerns a negative value and usually a big one)

a_{32} is the valuation of the risks and negative consequences that the blackmailed player assigns to the fact that he does not allow himself to be blackmailed and does not start to cooperate (in the given case this value is zero)

$a_{11}+ a_{21}+ a_{31}$ is the sum of all of the blackmailed player's payouts if he accepts the cooperation offer

$a_{12}+ a_{22}+ a_{32}$ is the sum of all of the blackmailed player's payouts if he does not accept the cooperation offer

Some notes to this:

1. The payout of the blackmailed player, no matter how he decides, consists of several components. As we have seen, this also holds in similar cases, i.e., not only when he faces the dilemma whether to allow himself to be blackmailed or not. When illustrating this in a corresponding schema, it is good to list the individual components under each other, so that also in the schema a brief characteristic can be assigned to them at least in one word. This makes the schema clearer. Anyone studying it will have a better idea of what is going on. The total sum of the payouts is then stated under the line.
2. In the model, the following applies in the logic of the matter:
   - if $a_{11}+ a_{21}+ a_{31} > a_{12}+ a_{22}+ a_{32}$, the player accepts the offer and allows himself to be blackmailed;
   - if $a_{11}+ a_{21}+ a_{31} < a_{12}+ a_{22}+ a_{32}$, the player does not accept the offer and does not allow himself to be blackmailed.
3. For the sake of completeness, we also include in the schema the values of those variables that are equal to zero in the given case, as they can have non-zero values in the schematic expression of similar situations.
4. A player can be mistaken when handling the said dilemma. The origin of the error can be in one or more of the following mistakes:

- The player is not aware of and does not consider ("does not include") an influence or consequence that plays a significant role in reality (in our case we stated three, there can be more in real situations).
- The valuation of an influence is inadequate (in the given case, consequences that will be faced by the blackmailed player if he accepts the cooperation offer and allows himself to be blackmailed usually tend to be significantly undervalued). This is usually the case because the player is unable to imagine how the game will continue to develop (we will deal with this later on).

5. One of the benefits of the presented schema is, among other things, that it makes it possible to identify, differentiate and describe individual cases of errors. In more complicated situations, where it is necessary to also take the other player's (the one who is doing the blackmailing) reaction into account, the cause of errors can also be an incorrect estimate of parameters according to which the other player is making his decisions. This is a considerably more complex case, which we will also discuss later on.

Let us demonstrate specific decision-making cases:

First example:
Let us assume the player gives the following valuation:

a_{11}	=	0		a_{12}	=	-7
a_{21}	=	+2		a_{22}	=	0
a_{31}	=	-5		a_{23}	=	0
Σ	=	-3	>	Σ	=	-7

The player accepts the cooperation offer because -3 > -7. But in this case the player could have overvalued the sanction that he would face for committing an act that makes it possible for him to be blackmailed, or conversely undervalued the negative consequences of allowing himself to be blackmailed.

Second example:

a_{11}	=	0		a_{12}	=	-5
a_{21}	=	+2		a_{22}	=	0
a_{31}	=	-8		a_{23}	=	0
Σ	=	-6	>	Σ	=	-5

Let us assume the player gives the following valuation:
The player does not accept the cooperation offer because -6 < -5. The player becomes aware of the negative consequences of what would occur if he allows himself to be blackmailed.

Third example:

a_{11}	=	0		a_{12}	=	-5
a_{21}	=	+4		a_{22}	=	0
a_{31}	=	-8		a_{23}	=	0
Σ	=	-4	>	Σ	=	-5

Let us assume the player gives the following valuation:
The player once again accepts the cooperation offer because -4 > -5. He is enticed by the relatively high reward for cooperating if he allows himself to be blackmailed. The high reward that he gets for cooperating could have ensued from the consideration (and precise valuations) of the second player. He set it exactly so that his offer would be accepted by the blackmailed player.

The examples listed above implicitly contain another important moment, which is the fact that the player must be able to adequately guess the future development. But in order to be able to do this, he must be capable of imagining what this future development will look like. In other words – if the corresponding valuations are to be adequate, he must be able to think several moves ahead. And for this we need to expand our original model considerably. There are a number of possibilities. What we will need for this, we have already in a certain sense inserted into our first schema, which described the basic dilemma of a player who is deciding whether or not to allow himself to be "drawn" into a structure based on mutual covering-up.

4 Tools of the Analysis of Games Based on Mutual Covering-up

As follows also from the previous methodological insert, the modeling of structures that are based on mutual covering-up is relatively difficult. We only have the first tools available to use and we are generalizing the first experiences from their use. We have to take into account the following, among other things:

1. That which one player (X_2) forces another player (X_1) to do depends on the contextual game type. If, for example, discriminating coalitions are being created, this can involve the support of the formation of such a coalition. Both players then face the following risk: a third person discovers that they are establishing a structure that is based on mutual covering-up, and this third person then faces a dilemma whether to spread news of his discovery or use the new information for the benefit of his game based on mutual covering-up. Etc.
2. In contextual games players accept various roles. (This can also be stated the other way around – contextual games select players into various roles and imprint certain typical game characteristics onto them.) If we want to put a payout matrix together, we have to take into account not only various types of contextual games that are based on mutual covering-up, but also the various roles that the players accept. Structures based on mutual covering-up can be very complex and developed as their development is subject to laws that are very similar to natural selection (only the most viable ones survive).
3. If the process of identifying the breaching of rules is institutionalized, the structures based on mutual covering-up must have the ability to install their players (who are subordinated to behavior based on mutual covering-up) into all institutional structures of this type.

In most games based on mutual covering-up, one player (the one generating the game that is based on mutual covering-up) controls (establishes power over, influences, blackmails) a larger number of players (those that were caught while breaching rules), the

roles of which usually supplement each other. In real situations, we also have to take into account the fact that systems have a more complex structure in the following two directions:

1. They are hierarchical systems; one of the systems consists of sub-systems at a lower hierarchical level.
2. They are systems in which institutions specializing in restricting rule breaching actions are formed.

Re: 1. If hierarchical systems are involved, it is significant who discovered the breaching of rules and where, i.e., what the relationship is between player X2 (the one who discovered the breaching of the rules and is abusing this fact) and X1 (the one who committed the breaching of rules and was caught). The following possibilities can occur here:
1.1. Both are in the same sub-system.
1.2. Each of them is in a different sub-system at the same level.
1.3. X_1 is in a sub-system, X_2 is in a hierarchically higher system.
1.4. X_2 is in a sub-system, X_1 is in a hierarchically higher system.
1.5. Both are in a hierarchically higher system.
(What can take place depends on the type of relations between the sub-systems and the system as well as among the sub-systems themselves.)

Re: 2. This concerns systems in which institutions specializing in restricting rule breaching actions are being formed. These are institutions in the following areas:
2.1. Identification of rule breaking.
2.2. Spreading of information about rule breaking.
2.3. Judging the degree of rule breaking and imposing sanctions.
2.4. Generating the creation and powers of institutions oriented on restricting actions that breach rules and generally accepted principles.

As has already been mentioned in the previous methodological insert, it is not completely obvious how to approach the problem and what should be the basis for grasping it. After the discovery of the phenomenon that was referred to as structures based on the mutual covering-up of the breaching of rules and generally accepted principles, our team did not know for several months how to describe and analyze it. Eventually, the path based on a certain symmetry between how a structure based on mutual covering-up is formed (which we presented – although only in the first approximation – in the part called Contextual games and identification of structures that are based on mutual covering-up), and how it could be disturbed proved to be promising. In other words, the symmetry between incorporating a player into a structure (binding him) where this player deals with a corresponding dilemma, and his eventual departure from the structure that is based on the mutual covering up (his release) where he also deals with a certain dilemma. Looking back, the discovery of the fact that the problem must be approached this way may appear trivial. But this is a frequent case when a theory is being formed.

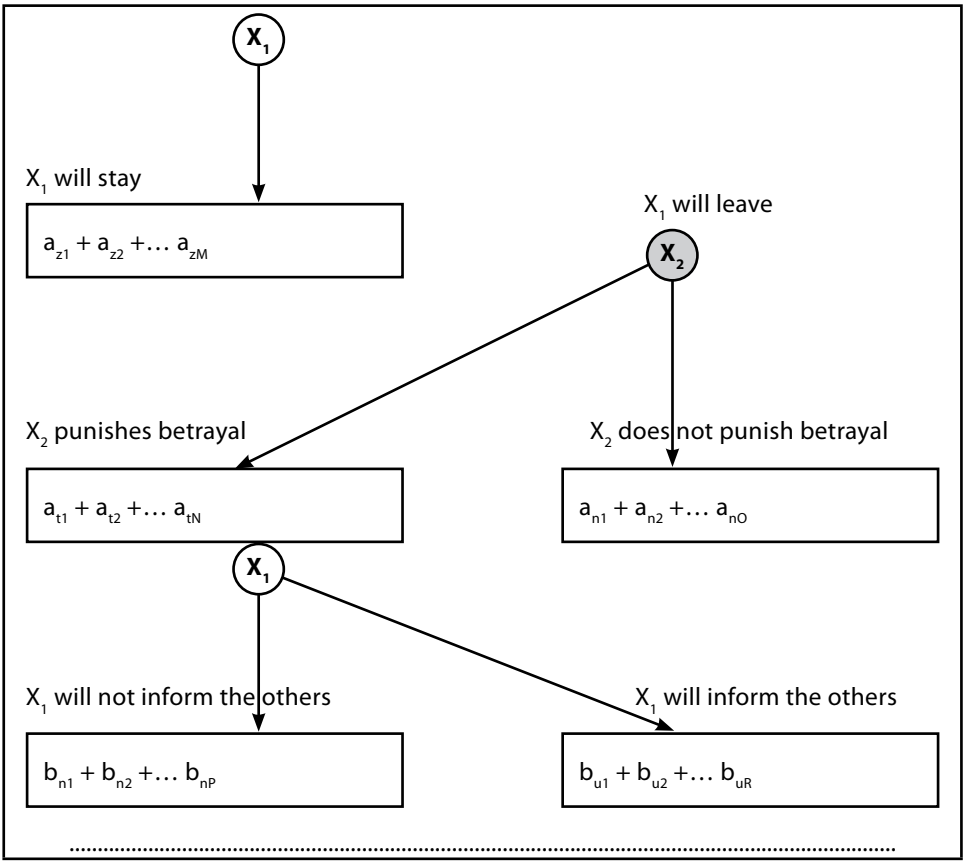
But contextual games are more complex in the case of the analysis of situations formed during the decision-making of a player about exiting a structure that is based on mutual covering-up. More precisely – if in a case where the player is considering the dilemma

whether to commit to a structure that is based on mutual covering-up he leaves the situation's further development unaccounted for (generally he is not aware of everything that he could come across and on what the further development will depend on), then in a case where he is deciding about the possibility of exiting a structure he is able and forced to see more into the future. If he also saw into the future in a similar way in a situation where he is dealing with the dilemma of committing to a structure, his assessment of the consequences would presumably be more accurate.

Let us therefore assume that one of the players faces the dilemma whether or not to exit a structure that is based on mutual covering-up, i.e., whether to allow himself to continue to be blackmailed and take actions that are against his beliefs as well as that bring him considerable risks, or to resist and possibly even fight against the corresponding structure and publish what he knows about it with the intention of eliminating it or weakening its influence. At the same time we will assume that the structure's core (i.e., the players who decide how the structure will behave), which up until now we have considered to be one of the players, itself consists of several players, which we will take into account (not immediately, but at a certain moment).

We will only consider the case where player X_1 in the first phase (that we will model) considers only the possibility of departing "quietly", i.e., not making public what he knows and not taking a stance against the structure. The structure based on mutual covering-up (which in our model will represent player X_2) then by contrast has the chance to penalize the player (punish for the fact that he has stopped subordinating) or allow the departure to take place without notice. At the same time it of course knows (and player X_1 – from whose position we are putting the model together – also knows this) that if it allows him to depart without notice (without a penalty), player X_1 will not make what he knows public. If it punishes him, the player can make this public. This will cause damage to the structure. But if it allows him to leave and does not punish him, other players forming the structure may find out about this and follow the first player. We can depict the situation as a game in an explicit form, as shown in the following illustration.

Figure 2: Schema describing a game between a certain subject and structure



Source: Created by the author.

Here:

$a_{z1} + a_{z2} + \dots a_{zM}$

is the sum of all (i.e., a total of M) impacts on player X_1 (valued in units that are homogenous with the valuation of all other cases), if player X_1 stays.

See the following table for more details (if we were to interpret the individual impacts like in Figure 1).

Table 4: Schema describing a game between a certain subject and structure

a_{z1} – risk of disclosure
a_{z2} – possibility of a career in the structure
a_{z3} – rewards ensuing from the possibility to function in the structure
…
a_{zM} – increase in structure tied-ness
$a_{z1} + a_{z2} + a_{z3} \dots a_{zM}$

Source: Created by the author.

In a similar way, we can also interpret and characterize other cases in more detail, e.g.,:
$bu_1 + bu_2 + \ldots bu_R$
is the sum of all (i.e., a total of R) impacts on the structure represented in the model by player X_2 (valued in units that are homogenous with the valuation of all other cases), if player X_1 departs (as this player sees it, respectively values it), the structure decides to punish him and player X1 starts to inform against the structure.

It is necessary to emphasize that in the given case the entire model is built based on the valuation of one player, in the given case of the player who is deciding whether to remain or not, i.e., player X_1. The second player (entire structure based on mutual covering-up) can see and assess the situation differently. But if the first player is deciding, he has no choice but to guess how the structure will react, respectively which parameters of the situation the structure will assess and how it will assess them. Of course player X_1 can be wrong as the structure may assess the situation differently.

There can be many impacts on individual players. For example, in the case of the player who is deciding, it can be the risk that the activity of the entire structure will be revealed and penalized, that the player will be forced to perform activities that go against his beliefs and represent growing risk for him, that by joining the structure he sacrifices his own other promising possibilities, etc. Similarly, there are also many possible impacts on the second player (i.e., the entire structure).

The game can continue on (as is suggested in Figure No 3). For example, player X_2, i.e., the corresponding structure that is based on mutual covering-up, can decide whether it will escalate the situation and punish the player that is informing against it more severely, even at the price of the risks that emerge as a result of this. At the same time, it is necessary to also take into account the fact that games between the player who wants to leave the structure and the structure itself induce other games inside the structure, e.g., between those who favor a more conciliatory solution and those who favor a more severe solution.

The case of a contextual game (of the induced type in the given case) that starts to be played inside a structure that is based on mutual covering-up among players favoring a more severe approach and the players favoring a more conciliatory approach must be modeled using means that we have not used yet.

Let us take the simplest case of a core of a structure that is based on mutual covering-up, which – if the creation of coalitions among the players that form the core is to play a role – will include three players in this case. With fewer players coalitions could not form. Let us assume that each player considers the following components via which he values each of the possible situations:

a_{T+} positive consequence of a harsher stance, which can be read, for example, as that the functioning of the structure is able to provide higher payouts to the core players

a_{T-} negative consequence of a harsher stance, which can be read, for example, as that the functioning of the structure does not enable the given core player to leave the structure without being penalized if he himself felt this need

a_{M+} positive consequence of a softer stance, which can be read, for example, as that a certain precedent is being created for the given core player to be able to leave the structure without being penalized if he himself felt this need

a_{M-} negative consequence of a softer stance, which can be read, for example, as that that the weakening of the structure threatens payouts that the given player from the structure's core will receive

a_{V+2} positive consequence for the corresponding core player if he is in a coalition that won the vote on the harsher or softer stance and this coalition is formed by two players (one is now in the opposition)

a_{V+3} positive consequence for the corresponding core player if he is in a coalition that won the vote on the harsher or softer stance and this coalition is formed by three players (no players are in the opposition)

a_{V-2} negative consequence for the corresponding core player if he is not in a coalition that won the vote on the harsher or softer stance and he now finds himself in the opposition

Let us also assume the following:

1. These are all relevant components that must be taken into account in the given situation.
2. The values of the listed components are the same in the case of all three players. (Regarding this let us note that this is a considerable simplification from the practical point of view.)
3. The payout of each player is equal to the sum of all compatible components.

In this case, situations can occur, the complete list of which can be presented by the following table, which contains $2^3 = 8$ different possibilities:

Table 5: The payout matrix of players that form the core of a structure based on mutual covering-up

X_{21}	X_{22}	X_{23}
T ($a_{T+} + a_{M-} + a_{V+3}$)	T ($a_{T+} + a_{M-} + a_{V+3}$)	T ($a_{T+} + a_{M-} + a_{V+3}$)
T ($a_{T+} + a_{M-} + a_{V+2}$)	T ($a_{T+} + a_{M-} + a_{V+2}$)	M ($a_{T+} + a_{M-} + a_{V-2}$)
T ($a_{T+} + a_{M-} + a_{V+2}$)	M ($a_{T+} + a_{M-} + a_{V-2}$)	T ($a_{T+} + a_{M-} + a_{V+2}$)
T ($a_{T-} + a_{M+} + a_{V-2}$)	M ($a_{T-} + a_{M+} + a_{V+2}$)	M ($a_{T-} + a_{M+} + a_{V+2}$)
M ($a_{T+} + a_{M-} + a_{V-2}$)	T ($a_{T+} + a_{M-} + a_{V+2}$)	T ($a_{T+} + a_{M-} + a_{V+2}$)
M ($a_{T-} + a_{M+} + a_{V+2}$)	T ($a_{T-} + a_{M+} + a_{V-2}$)	M ($a_{T-} + a_{M+} + a_{V+2}$)
M ($a_{T-} + a_{M+} + a_{V+2}$)	M ($a_{T-} + a_{M+} + a_{V+2}$)	T ($a_{T-} + a_{M+} + a_{V-2}$)
M ($a_{T-} + a_{M+} + a_{V+3}$)	M ($a_{T-} + a_{M+} + a_{V+3}$)	M ($a_{T-} + a_{M+} + a_{V+3}$)

Source: Create by the author.

Here:

X_{21}, X_{22}, X_{23}	are players that form the core of the structure that is based on mutual covering-up
M, T	soft or harsh version of the stance that the corresponding player will take
~~T~~, ~~M~~	the corresponding alternative of the stance that did not receive support (the carrier of which finds himself outside of the winning coalition) has been crossed out
$(a_{T+} + a_{M-} + a_{V+3})$	one of the cases of the presentation of components from which the total payout of the corresponding player in the situation that occurred consists

The schema presented in table no. 3 is not nearly as complicated as could appear at the first glance. We will demonstrate this on an example. Let:

Valuation components:	Comment to this:
$a_{T+} = 8$	8 is greater than 5, i.e., in the given case the structure is on the rise, players value the proceeds from the structure more than the possibility of leaving
$a_{T-} = -3$	-3 is less than -4, i.e., the players relatively highly value the risk that the possibility of departing the structure will decrease
$a_{M+} = 5$	
$a_{M-} = -4$	
$a_{V+2} = 7$	the players assign a relatively high payout to the state where they find themselves in the winning coalition and one player ends up outside of it
$a_{V+3} = 0$	if all three vote the same way, the payout valuing the change of position has a zero value
$a_{V-2} = -10$	the players assign a relatively high negative payout to being left outside the winning structure

If we establish these valuations of the individual components, we get the following values:

Table 6: Payout matrix of the core players of a structure that is based on mutual covering-up, with specific values

X_{21}	X_{22}	X_{23}
T (8 + -4 + 0 = **4**)	T (8 + -4 + 0 = **4**)	T (8 + -4 + 0 = **4**)
T (8 + -4 + 7 = **11**)	T (8 + -4 + 7 = **11**)	M (8 + -4 + -10 = **-6**)
T (8 + -4 + 7 = **11**)	M (8 + -4 + -10 = **-6**)	T (8 + -4 + 7 = **11**)
T (-3 + 5 + -10 = **-8**)	M (-3 + 5 + 7 = **9**)	M (-3 + 5 + 7 = **9**)
M (8 + -4 + -10 = **-6**)	T (8 + -4 + 7 = **11**)	T (8 + -4 + 7 = **11**)
M (-3 + 5 + 7 = **9**)	T (-3 + 5 + -10 = **-8**)	M (-3 + 5 + 7 = **9**)
M (-3 + 5 + 7 = **9**)	M (-3 + 5 + 7 = **9**)	T (-3 + 5 + -10 = **-8**)
M (-3 + 5 + 0= **2**)	M (-3 + 5 + 0= **2**)	M (-3 + 5 + 0= **2**)

Source: Created by the author.

The resulting values are marked here in bold. For the sake of clarity, the next table indicates only the total size of the payouts.

Table 7: Payout matrix of the core players of a structure that is based on mutual covering-up, with specific values (just the outcome)

X_{21}	X_{22}	X_{23}
T (**4**)	T (**4**)	T (**4**)
T (**11**)	T (**11**)	~~M~~ (**-6**)
T (**11**)	~~M~~ (**-6**)	T (**11**)
~~T~~ (**-8**)	M (**9**)	M (**9**)
~~M~~ (**-6**)	T (**11**)	T (**11**)
M (**9**)	~~T~~ (**-8**)	M (**9**)
M (**9**)	M (**9**)	~~T~~ (**-8**)
M (**2**)	M (**2**)	M (**2**)

Source: Created by the author.

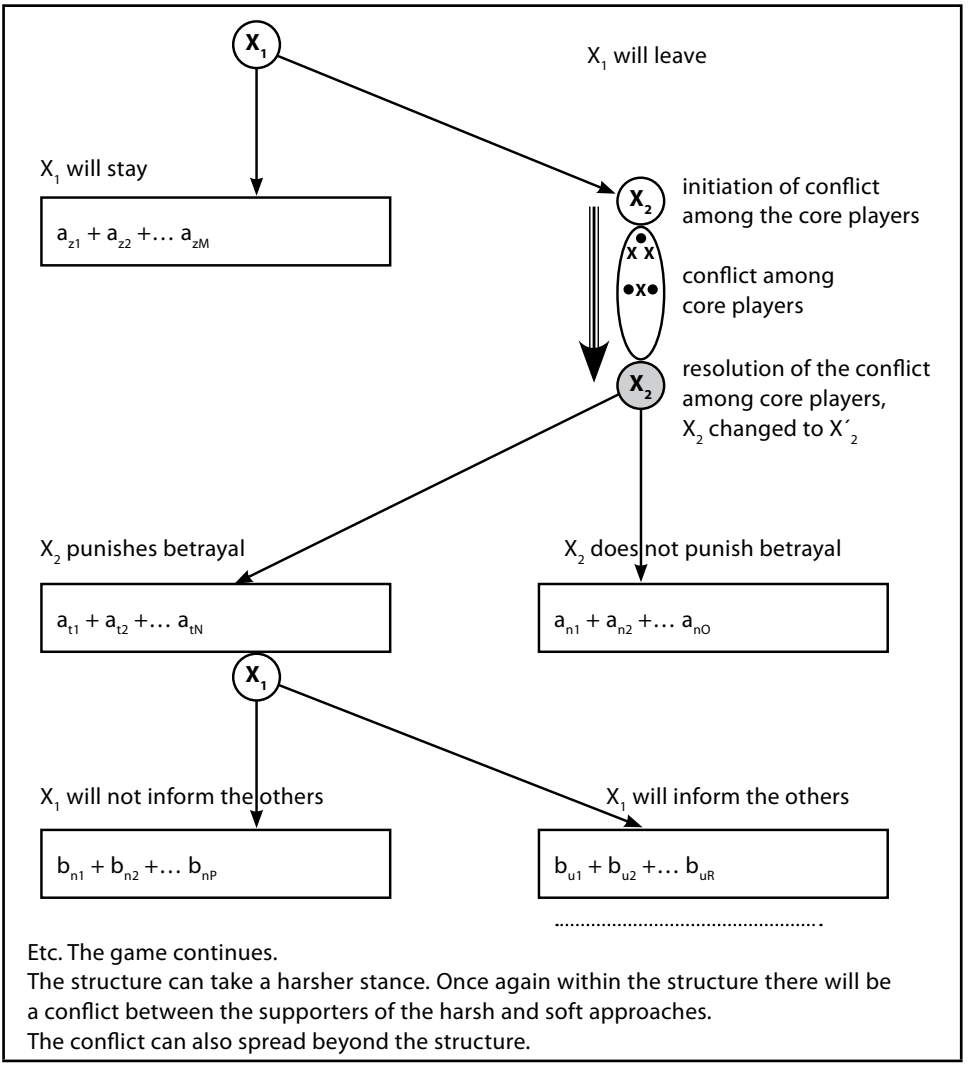
The table shows that with the given input data, each player:

1. Has an average payout of 4 when supporting the harsh approach, 3.5 when supporting the soft approach.
2. Has a higher payout three times and a lower payout just one time when supporting the harsh approach.
3. Can have the lowest payout (-8) when supporting the harsh approach, whereas when supporting the soft approach the least he can get is -6.

In general, it is possible to say that each player can count on the fact that the other players in the given case will support the harsh approach and thus for him it is also most advantageous to support the harsh approach. A more detailed analysis of decision-making under different parameters is beyond the scope of the presented contribution. We will only note some aspects:

- If more than three players are involved, we have to take into account their powers of influence if they find themselves in the minority during the decision-making. In our case, we used a negative payout component instead of decreasing the power of influence.
- Under different parameters the tendency towards orthodoxy usually wins, which can also be understood as group proving loyalty and becoming convinced of it.
- The analysis of decision-making of the type stated above shows certain similar attributes to what is being examined within the redistribution systems theory. This suggests that presumably there exists a more general model, of which the cases handled by the redistribution systems theory, the case that we have just dealt with, and probably other cases as well are specific cases.

Figure 3: Schema describing a game between a certain subject and a structure



Source: Created by the author.

Worth mentioning is also the possibility of expanding the conflict beyond the given structure that is based on mutual covering-up, as this structure is weakened by the conflict, of which other structures that compete with it in the given social space could take advantage and weaken its position or even eliminate it. On the other hand, by doing this they could also create a precedent for their own players who are pondering the idea of leaving the structure. That is why the decision-making of the structure's core does not have to be unanimous, and also in this case one can assume a certain tendency towards orthodoxy. Moreover, individual structures that are based on mutual covering-up can be mutually penetrated by cross-coalitions, i.e., coalitions that form among various structures that are based on mutual covering-up.

Conclusions and an Outline of Future Research Directions

A number of conclusions as well as future research directions ensue from the presented results. Let us present the most important ones:

1. When analyzing what is taking place in society, it is good to constantly clarify the notion that is based on differentiating what "is visible" and what "is not visible" (i.e., remains hidden and keeps hiding)
2. Analysis of what can be seen enables us to reveal (identify and analyze) more and more layers of what was originally not visible.
3. Tools from the redistribution systems theory can be used to model and analyze what can be seen.
4. When modeling and analyzing what is hiding, the concept of structures that are based on mutual covering-up of the breaching of rules and generally accepted principles – which also includes the concept of this phenomenon and tools offered by game theory – proves to be very important.
5. The palette of instruments provided by game theory for analysis of what can be seen, as well as of what cannot be seen immediately, is very rich.
6. An important part of research in this particular area is moving from partial concepts to an integrated consistent theory that relies on a mathematical apparatus.
7. For most tasks it is not possible to immediately use the existing game theory tools because these were usually developed for another purpose; however, some time-tested approaches and means of this theory can be modified and applied to this area.
8. The results and experience up until now suggest that the program of expressing social events via mathematical means within the context described above can be successful, and that much more can be accomplished in the given direction than could appear at the first glance.

The progress of our team that deals with the issues mentioned above can be followed "on-line" at www.vsfs.cz/?id=1046, where ongoing results as well as an archive of source materials for individual seminars since October 2003 are continuously available. The modeling of structures that are based on the mutual covering-up of the breaching of rules also has considerable practical importance because, among other reasons, it shows why the results attained so far in the battle against corruption and similar social phenomena are – to put it mildly – unsatisfactory.

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