The Efficiency of GARCH Models in Realizing Value at Risk Estimates
Účinnost GARCH modelů při realizaci odhadů Value at Risk

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Abstract
Market risk is an important type of financial risk that is usually caused by price fluctuations in financial markets. One determinant of market risk comprises Value at Risk (VaR), which is defined as the maximum loss that can be achieved within a certain time horizon and at a given reliability level. The aim of the article is to determine the importance of selecting conditional volatility model within the parametric and semi-parametric approach for VaR estimation. The results ascertained show that the application of these models tends to provide more accurate predictions of actual losses as compared to traditional approaches to VaR estimates. Overall, the application of conditional volatility models ensures that VaR estimates are more flexible to adapt to changing market conditions – especially in the periods associated with higher return volatility. Furthermore, the results show that the differences between individual models of contingent volatility are primarily determined by selecting the specific distribution of the standardized residue series.

Keywords
Value at Risk, GARCH models, distribution of standardized residues, extreme values theory

JEL Codes
C22, C52, C53, G15

DOI
http://dx.doi.org/10.37355/acta-2020/1-03

Abstrakt
Tržní riziko je důležitým typem finančního rizika, které je zpravidla způsobeno cenovými pohyby na finančních trzích. Jednou z měr tržního rizika je Value at Risk (VaR), jež je definována jako maximální ztráta, které lze dosáhnout v určitém časovém horizontu a při dané úrovni spolehlivosti. Cílem článku je určit důležitost volby modelu podmíněné volatility v rámci parametrického a semiparametrického přístupu pro odhad VaR. Zjištěné výsledky ukazují, že aplikace těchto modelů má tendenci poskytovat přesnější předpovědi skutečných ztrát, a to ve srovnání s tradičními přístupy pro odhad VaR. Celkově aplikace modelů podmíněné volatility zajišťuje, že získané odhady VaR se daleko flexibilněji přizpůsobují měnícím se tržním podmínkám – především v obdobích spojených s vyšší volatilitou výnosů. Výsledky dále ukazují, že rozdíly mezi jednotlivými modely podmíněné volatility jsou primárně dány výběrem konkrétních rozdělení standardizovaných reziduí výnosových řad.

Klíčová slova
Value at Risk, GARCH modely, rozdělení standardizovaných reziduí, teorie extrémních hodnot

Introduction

The growth in the importance of the market risk management process is strongly motivated by increased volatility in the financial markets, especially over the past few decades, which was reflected in a stronger effort to search for appropriate risk measurement approaches. The most well-known risk measure comprises Value at Risk (VaR), which is defined as the maximum loss that can be achieved within a certain time horizon and at a given reliability level. VaR was introduced in 1994 as a risk management method under the RiskMetrics system of the J.P. Morgan bank. Theoretical knowledge about VaR is provided by, for example, by Jorion (2007) or Alexander (2008). Later VaR began to be criticized for the lack of sub-additivity and convexity, see, for example, Artzner et al. (1999), resulting in the measure called Expected Shortfall (ES). *t*-day ES with the reliability level α represents an average loss exceeding VaR with the same time horizon and the same reliability level – hence the next ES denomination as a conditional or average VaR. Therefore, it is clear from the specification mentioned that the ES estimate primarily depends on the VaR's quality of estimation. In other words, although we focus on the ES estimate, it is still necessary to carefully choose the VaR methodological apparatus.

As a part of the parametric approaches to VaR estimation, the choice of a suitable distribution of the returns of individual risk factors that affect the price of individual assets or the entire portfolio is pivotal. The initial assumption introduced in RiskMetrics was that these returns had a normal or Gaussian distribution. However, this assumption has proved to be unrealistic, see, for example, Christoffersen and Diebold (2000), Pafka and Kondor (2001) or Bauwens and Laurent (2005). A certain group of researchers is of the opinion that it is not necessary to know the complete distribution of the analysed financial returns, but only their tails, i.e., only returns (or losses), which are very unlikely to happen. It, thus, concerns extreme losses or extreme incomes. For this purpose, the so-called Extreme Values Theory (EVT), which is the basic representative of a group of semi-parametric approaches for VaR estimation, has become very popular. One of the pioneers of the EVT application in measuring market risks was McNeil (1997) who compared EVR-based VaR estimates with other popular approaches and found out that EVT delivers the best results in measuring the market risk of the German stock index DAX 30. Embrechts et al. (1999); Longin and Solnik (2001); Gilli and Kellezi (2006); Diamandis et al. (2011), Radivojevic et al. (2016), Sowdagur and Narsoo (2017) belong to the other works.

In the following years, the application of conditional volatility models in market risks measurement appeared significant. Therefore, there is a view that the instability of the distribution parameters over time also affects the distortion of normalcy, namely that volatility of returns changes over time. For example, Christoffersen et al. (2001) compared VaR performance in using the EWMA and GARCH (1,1) volatility models with the S&P 500 stock index. The results showed that the VaR accuracy estimates at 95% and 99%

reliability levels based on GARCH (1,1) overshadows the second rated model. Engle and Manganelli (2004) reach a similar conclusion, showing a significant inaccuracy of EWMA-based estimates, i.e., the original RiskMetrics. The authors mentioned see the presence of extreme returns in the analysed data series as the reason for the failure of these models.

In order to capture the influence of heavy tails of financial returns, Bams et al. (2005) and Hartz et al. (2006) applied VaR models based on GARCH and using different probability distributions. Their works suggest that the performance of GARCH with Student distribution is more efficient than GED based errors distribution. So and Yu (2006) examined the performance of seven different types of GARCH models within VaR estimates of stock indexes and exchange rates. Overall, they conclude that Student-based distribution models give a more accurate estimate of 1% VaR than normal distribution. From the most recent works, for example, Mutu et al (2011) examined the performance of VaR models using the historical simulation, the extreme values theory (EVT) and the GARCH volatility model and exponentially weighted moving average (EWMA) model for five CEE and East European indexes BET, PX50, BUX, SOFIX and WIG20. The authors focused particularly on the crisis period around 2009 and show that VaR application with EVT and GARCH overcomes other approaches. Abad et al. (2016) examine the performance of VaR models with EVT access. In the volatility modelling, various modifications of the GARCH model are used together with both the normal and the Student distribution. Comparison of models' performance is made both during a volatile season and in case of periods with more volatile financial returns. The results show that the best performance (for both periods studied) is achieved by applying an asymmetric GARCH model with Student distribution.

The aim of the article is to determine the importance of choosing conditional volatility model within the parametric and semi-parametric approach to VaR estimation. Motivation for the stated goal is also based on the current popularity of non-parametric historical simulation for VaR, both in financial and non-financial institutions. The application of historical simulation is simple, but its disadvantage comprises too much attachment to the historical development of the analysed returns. The rest of the text is arranged in four chapters. The first chapter presents the methodology used, namely the principle of estimation of Value at Risk, the modelling of the conditional average value and the volatility and subsequently the way of testing the quality of the obtained estimates. The second chapter includes the data used, including their basic characteristics. The results are presented in the third chapter with the discussion in the fourth chapter.

Methodology

Value at Risk

VaR represents the worst possible loss that occurs over a certain period of time at a given reliability level. VaR therefore gives risk managers information what maximum funds the institution might lose at some future moment and with a certain likelihood. Since its introduction in the 1990s VaR has become a standard measure of risk. We consider a random variable r whose values represent financial returns. Then the VaR variable with the realiability α, VaR_{α} is determined by (1-α)-variable division of random variable r_t, i.e.,

$$VaR_{\alpha} = q_{1-\alpha}(r_t). \tag{1}$$

Several approaches are used to estimate VaRs, with output being the structure of the revenue distribution of a given instrument or the entire portfolio. In the case of parametric approaches, these are some of the standard divisions, most often normal. In this case, it applies for the VaR estimate that

$$VaR_{\alpha} = \mu + \sigma\Phi_{1-\alpha}^{-1}, \tag{2}$$

where μ, or more precisely σ is the median value, or more precisely the standard deviation of the return series and $\Phi_{1-\alpha}^{-1}$ represents the (1-α) variable of the normalized normal distribution, i.e., the normal distribution with zero median value and unit scatter. In addition to the parametric approach, a nonparametric historical simulation can also be used, which employs the so-called empirical distribution of returns, i.e., it uses a historical sample of the last few observed analyses of returns. The specific VaR value is then determined as the respective magnitude of this empirical distribution.

One of the critical factors in the VaR estimation is the density of the distribution of financial revenues, especially in the tail area. In order to increase the accuracy of the VaR, the so-called extreme value theory is used (EVT), which directly selects some extreme values from the available sample to best match the empirical tail distribution, instead of estimating the entire distribution with the entire spectrum of samples. An approach based on the theory of extreme values focuses on a certain limited distribution of extreme returns that is essentially independent of the distribution of revenues itself. For the realization of the EVT approach, the so-called limit crossing method is used in the area of finance (POT), which is based on the use of the generalized Pareto division (GPD).

For the purpose of the POT performance, let us consider the random variables $r_1, r_2, \ldots, r_n$, representing the financial returns. Among these variables we choose threshold value *v* and examine all variables exceeding u. Let us title the values of these variables $g1, \ldots, g_N$, where $g_i = r_i - u > 0$ and *N* is the number of returns greater than *u*. Now let us define the distribution function $F_u(x)$ of the distribution values exceeding u as

$$F_u(x) = P(r - u < g | r > u) = \frac{F(g+u) - F(u)}{1 - F(u)}. \tag{3}$$

However, given that for a large u N is generally very small, i.e., a very small number of values exceed a given threshold the estimate of F_u (x) could represent a rather complicated problem. Therefore, for a large u instead of F_u (x) its easier estimated approximation can be used. On the basis of the Balkema and De Haan theses, see Balkema and De Haan (1974) apply to $u \to \infty$

$$F_u(x) \approx G_{\xi,\sigma}(g), \tag{4}$$

for $y \in [0, r_F - u]$, if $\xi \geq 0$, where r_F is the right final point of returns distribution, and $y \in \left[0, -\frac{\xi}{\sigma}\right]$, if $\xi < 0$. $G_{\xi,\sigma}$ is the so-called generalized Pareto division (GPD). Parameter ξ indicates the power of GPD tails – in particular the larger the ξ value, the heavier the GPD tails. Thus, for modelling financial returns GPD is more suitable where $\xi \geq 0$. Now the distribution function can be expressed as follows

$$F(r) = F(g + u) = [1 - F(u)]G_{\xi,\sigma}(g) + F(u). \tag{5}$$

The only thing that is left to estimate is $F(u)$. For this purpose we will place $F(u) = \frac{n-N}{n}$ and subsequently we get

$$F(r) = \frac{N}{n}\left(1 - \left(1 + \frac{\xi}{\sigma}(r - u)\right)^{-\frac{1}{\xi}}\right) + \left(1 - \frac{N}{n}\right), \tag{6}$$

and by subsequent adjustment we add

$$F(r) = 1 - \frac{N}{n}\left(1 + \frac{\xi}{\sigma}(r - u)\right)^{-\frac{1}{\xi}}. \tag{7}$$

The parameters of the distribution function (7) can be estimated by several approaches, of which the most popular is the maximum assurance method to be used in this text. For the VaR estimate it then applies that

$$VaR_\alpha = u + \frac{\sigma}{\xi}\left(\left(\frac{n}{N}\alpha\right)^{-\xi} - 1\right). \tag{8}$$

Modelling of the conditioned median value and volatility

A simplistic assumption within the analysis of financial time series is that financial returns are independent, equally distributed random variables with zero median value and constant volatility. However, this assumption is unrealistic in the vast majority of cases. In case of its deletion, returns can be modelled through the following equation

$$r_t = \mu_t + \sigma_t z_t, \tag{9}$$

where μ_t is the median value dependent on time t (we refer to the so-called conditional median value) and σ_t is the time changing volatility (the so-called conditional volatility). Further, z_t represents a random variable with identically and independently divided values, assuming zero median value and unit scatter.

In order to model the conditional median value of financial returns, the autoregressive (AR) model is most commonly used. It is a simple model of a stationary time series designed by Box and Jenkins (1970). The aim of this model is to remove linear dependencies from the time series to obtain residues that are not mutually correlated. The conditional median value μ_t can be expressed as AR (m) of the model as follows

$$\mu_t = \mu + \sum_{i=1}^{m} \phi_i r_{t-i}, \tag{10}$$

where μ is the unconditional median value of the time series $\Phi_1,\ldots,\Phi_m$ are estimated autoregressive coefficients.

Different approaches can be used for the purpose of modelling σ_t. In particular, we refer to the frequently used EWMA model in the literature, as well as several representatives of the GARCH model class. EWMA represents the easiest option for conditional volatility modelling and is based on the use of the smoothing parameter λ, which determines the rate of smoothing the effect of earlier return observations. Specifically, for a sufficiently large *n*, the EWMA can be expressed as follows

$$\sigma_t^2 = (1-\lambda) \sum_{i=0}^{n-1} \lambda^i (\varepsilon_{t-i})^2. \tag{11}$$

The EWMA model was used in the known RiskMetrics approach for Value at Risk estimate, designed by J. P. Morgan bank. For the smoothing parameter, RiskMetrics considers the value of λ=0,94.

The basis for GARCH models comprises the conditional heteroskedasticity model (ARCH), where the conditional volatility in process (1) can be expressed as follows

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i (\varepsilon_{t-i})^2, \tag{12}$$

where $\alpha 0>0$ and $\alpha_i \geq 0$ for $i=1,\ldots,q$. Volatility is, therefore, presented as a linear combination of *q* residual squares within the ARCH (q) model. The problem of the ARCH model is its inability to identify the autocorrelation structure of conditional volatility, see Brooks (2008). The solution is the generalized model of conditional heteroskedasticity (GARCH). In general, the GARCH (*p,q*) model can be defined as follows

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i (\varepsilon_{t-i})^2 + \sum_{i=1}^{p} \beta_i (\sigma_{t-i})^2, \tag{13}$$

where $\alpha_0>0$, $\alpha_i,\beta_i \geq 0$ for $i=1,\ldots,q$, or more precisely $i=1,\ldots,p$. Thus, the conditional volatility is defined in the GARCH model by means of a linear combination of residual squares (as well as by ARCH) and by using historical values of conditional volatility itself.

The models so far mentioned do not fully reflect the nature of the volatility of the revenue time series. Although they are well characterized by volatility clustering, the problem arises from their inability to model the asymmetric effects in the model, namely the leverage effect. The leverage effect is discussed in the case when the "wanted development" of the time series is negatively correlated with changes in volatility, in the sense that volatility decreases, for example, in the growth of returns, or more precisely the decrease of losses – i.e., in case of the "good" development of the series, and on the other hand, the increase in volatility occurs when the returns decline, or more precisely the losses increase – i.e. in the "bad" development of the series. In other words, the previous models depend on the squares of residues, and therefore the effect caused by the positive shock is the same as in the case of negative shock. In order to capture the leverage effect, several innovations of the existing GARCH models have been proposed, whereas the most commonly used comprise the GJR-GARCH models proposed by Glosten et al. (1993). For conditional scattering estimated by GJR-GARCH (p, q) model the following applies

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{p}\left(\alpha_i |e_{t-1}|^{\delta} + \gamma_i |e_{t-1}|^{\delta} I_{t-i}^{-}\right) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta}, I_{t-i}^{-} = \tag{14}$$

$$= \begin{cases} 1, \text{for} e_{t-i} < 0 \\ 0, \text{other way} \end{cases}$$

where $\delta \geq 1$, $\alpha_0 > 0$, $\alpha_i \geq 0$, $-1 \leq \gamma_i \leq 1$, $\beta_j \geq 0$, pro $i=1,\ldots,p$, or more precisely $j=1,\ldots,q$ and further for stationarity of the model the following condition $\frac{\gamma+\alpha}{2} + \beta < 1$ needs to be met. The coefficient γ is just an indicator of the leverage effect. Of the model group (14), the model using δ = 2 is most often used. It is often referred to as GJR-GARCH for its frequent use. Another widely used class representative of the above models is the TGARCH model, referred to as the threshold model, which results from equation (14) by laying δ = 1. Parameters of all three GARCH models used are estimated by the maximal assurance method, which provides asymptotically effective estimates of the search parameters.

For the VaR estimation with the reliability of α realized by applying the above models the following applies

$$VaR_{\alpha} = \mu_t + \sigma_t q_{1-\alpha}, \tag{15}$$

where $q_{1-\alpha}$ (1-α) is the (1-α) quantum of the used random variable z_t, whereas for the EWMA model we consider z_t, as a random variable with a normalized normal *N* (0,1) distribution. For the class of GARCH models, in addition to the mentioned distribution the Student's and tilted Student's distribution is also used.

Testing the quality of the obtained estimates

VaR values are obtained at a reliability level of 95% and 99%. As a standard, the ratio of the relative performance of VaR, $\hat{a}$ and the α-variable considered is used as a benchmark for comparing and evaluating the approaches used, where $\hat{a}$ represents the ratio of the number of estimates underestimating the actual realized loss and the number of all estimates obtained. The $\hat{a}$ value then corresponds to the theoretical relative error rate of the given model, namely for 95% reliability it is $\alpha = 0.05$ and for 99% reliability then $\alpha = 0.01$. Preference is given to models, for which this ratio is close to 1.

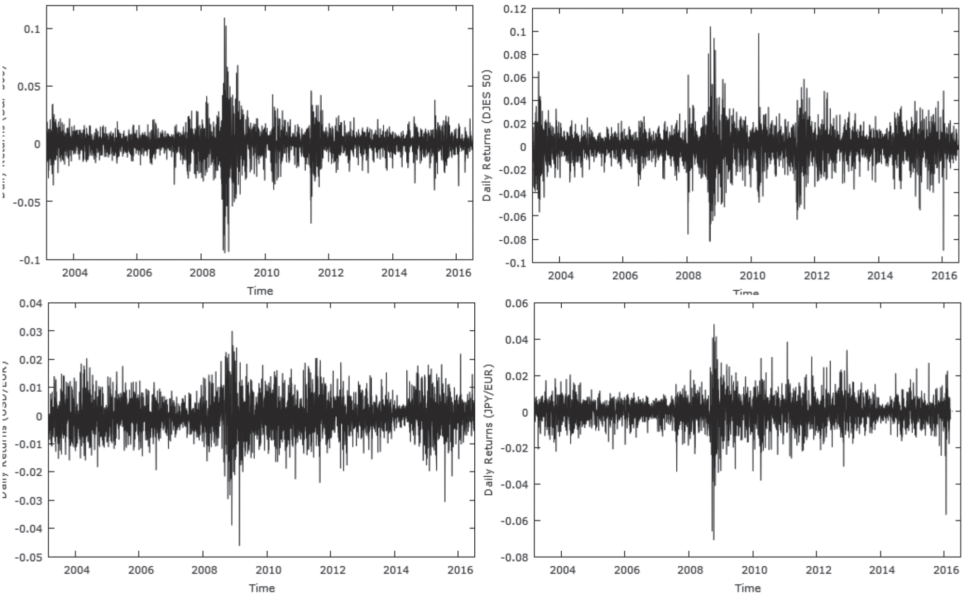
In addition to this informal back testing method, we use two formal approaches to determine the quality of the VaR estimates, namely the unconditional Kupiec (UC) test, which tests whether the model's failure rate corresponds to VaR's significance, see Kupiec (1995). In other words, it is about whether the deviation of $\frac{\hat{\alpha}}{\alpha}$ from value 1 is statistically significant. Higher number of failures, i.e., $\frac{\hat{\alpha}}{\alpha}>1$, are identified by the test as underestimating the risk, while a lower failure rate, i.e., $\frac{\widehat{\alpha}}{\alpha}$α<1, is considered unnecessary risk overestimation. The disadvantage of this test is its inability to determine whether failures are random, i.e., uncorrelated over time. This problem can be solved by applying Christoffersen's conditional test, see Alexander (2008). By combining both tests, we get the so-called Mixed Kupiec (CC) test, which can be considered as the overall model quality indicator.

Data and its empirical analysis

Four financial data samples are used in this work, namely two stock indexes, the Dow Jones Euro Stoxx 50, which measures the performance of 50 stock titles of leading companies in the Eurozone and the S&P 500 stock index, consisting of 500 stock titles of US companies. And further two exchange rates, especially USD/EUR and JPY/EUR. The data sample covers the period from 1 January 2003 to 31 August 2016 and dates are expressed in the form of daily logarithmic returns.

Figure 1 shows the evolution of daily returns of all monitored variables. Within all four ranges, an average zero return is apparent, as is shown in Table 1.

Figure 1: Daily logarithmic returns of stock indexes and exchange rates



Source: Author's construction

As noted above, most financial series are subject to a conditional median value that changes over time and volatility. From the charts in Figure 1 it can be concluded that these phenomena occur within the analysed series. Statistical tests are used for verification, namely the Ljung-Box Q test and the ARCH test; see, for example, Brooks (2008). In case of three of four monitored series, we reject the Q test application at 1% of the significance of the hypothesis on the separation independence – see Table 1, where value 1 indicates the assumption of an alternative hypothesis about the breach of the independence of the returns over time, and the value 0 then confirms the tested independence. In case of the European stock index, the 1% level of autocorrelation was not confirmed. Furthermore, using the ARCH test in all rows, we reject the zero hypothesis about the absence of ARCH effects, so the used return series are burdened with time-varying conditional volatility.

For the purposes of further analysis of the observed returns, table 1 presents basic descriptive statistics. The minimum and maximum values are relatively far from the averages, which confirms the occurrence of extreme events during the time period under review. The value of the standard deviation of returns, or their volatility, is slightly higher than the exchange rates for stock indexes. Additionally, the skewness of all returns is negative. This fact indicates that the presence of extreme values in the left-hand tails of the analysed distributions is more frequent in the presence of extreme values in the right tails. The standardized kurtosis value is greater than 0, indicating that the revenue distribution is sparser than the normal distribution, most notably the Japanese Yen exchange rate. This means that most of the returns are concentrated around the mean, but there are more distant observations.

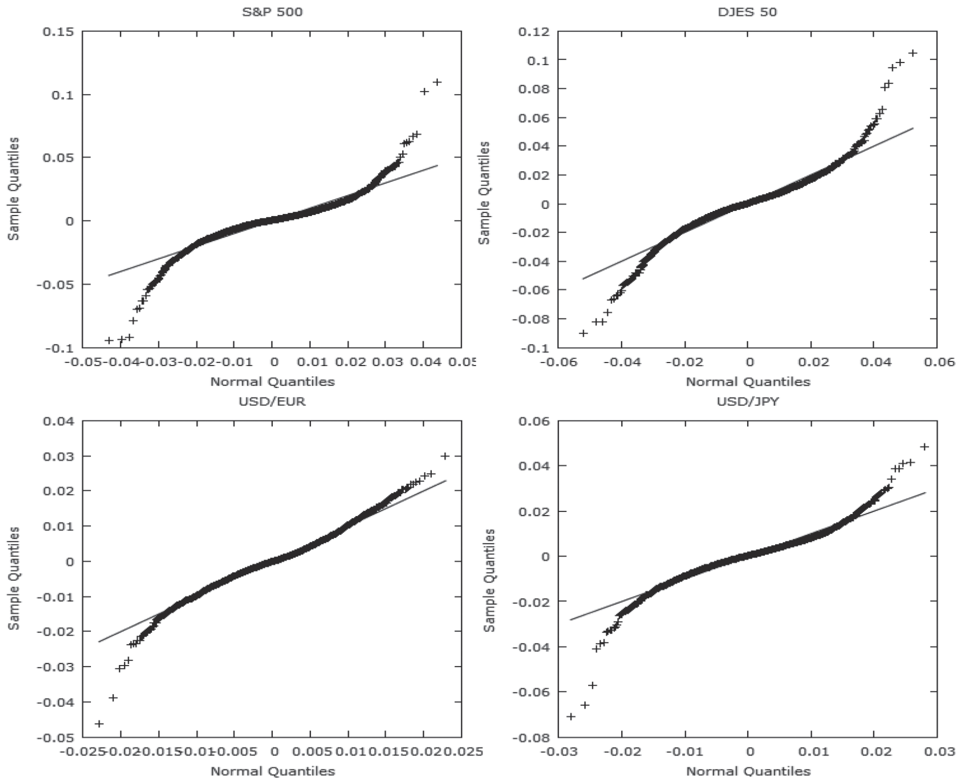
These stated characteristics, such as negative obliquity and higher sharpness, are typical features of financial returns and carry the problem of the so-called heavy tails. The point is that there are extreme losses (negative returns) in these ranges, which result in the distribution of returns not being Gaussian or normal. In order to verify that the normality of returns is actually violated, we use the Q-Q quantity graphs shown in Figure 2. These are charts, in which we enter against each other the theoretical distribution – in our case, the normal distribution and the quantitative empirical distribution.

Table 1: Basic characteristics of daily returns

	SP 500	DJES	USD/EUR	JPY/EUR
Mean	0,0001	0,0001	0,0001	-0,0001
Median	0,0007	0,0003	0,0000	0,0002
Min	-0,0569	-0,0764	-0,0410	-0,0662
Max	0,0609	0,0729	0,0273	0,0435
Standard Deviation	0,0110	0,0140	0,0064	0,0082
Skewness	-0,1980	-0,1262	-0,1657	-0,4210
Kurtosis	3,3806	3,9978	2,4356	6,2345
Jarque-Bera	1	1	1	1
Q test	1	0	1	1
ARCH test	1	1	1	1

Source: Author's own processing

Figure 2: Quantitative (Q-Q) graphs of daily returns



Source: Author's construction

It is clear from the graphs that the drawn points are S-shaped, which is a typical manifestation of the existence of heavy tails. Therefore, we cannot talk about the normality of the monitored returns. This fact is also confirmed by the application of the Jarque-Bera test, which rejects the hypothesis on the normal distribution of the analysed data at 1% materiality level.

Results

In this section, we focus in detail on comparing the predictive performance of the the above-described approaches to VaR estimates, with particular attention being paid to the application of three GARCH models in the selection of different types of distribution. As specific approaches to the VaR estimation, or more precisely ES both traditional approaches represented by parametric variation-covariance method, parametric approach used in RiskMetrics, i.e. with EWMA application and nonparametric historical simulation are used. As a part of the historical simulation, we consider a historical sample covering the last 250 observations (HS250), as well as a historical sample involving observations corresponding to the last two business years, i.e., the range of 500 recent earnings (HS500). The choice of these ranges corresponds to the methodology of historical simulation applied by most banks operating in the Czech Republic.

GARCH models are applied within the parametric approach to the VaR estimation where AR (1)-GARCH (1,1), AR(1)-GJR-GARCH (1,1) and AR(1)-TGARCH(1,1) models are used to model the conditional median and conditional volatility (1)-TGARCH (1,1) models, where for independent and equally divided residues z_t, from equation (9), we consider the normal, Student's and tilted Student's distribution. In addition, GARCH models are used within the semi-parametric approach to VaR estimation, namely AR (1)-GARCH (1,1)-EVT, AR(1)-GJR-GARCH (1,1)-EVT and AR(1)-TGARCH (1,1)-EVT.

The values in table 2 represent $\hat{a}/\alpha$ ratios, where $\hat{a}$ represents the percentage of one-day estimates underestimating the actual state and α then represents the model of the assumed percentage of underestimation given by the VaR's reliability; we consider specifically that α = 0,05 and 0,01. For ratios marked bold, it applies that according to the UC test at 5% level of significance, their $\hat{a}$ is statistically significantly different from $\hat{a}$. Ratios not highlighted, on the other hand, meet the UC test. Bold framing for a given series of returns is the best model with a ratio approaching 1. For example, the α /0,01≈2,5 value for VC method of estimating VaR returns of the European index means that the output of this method is 2,5 times more erroneous estimates than expected. Specifically, the given method implements 44 erroneous predictions, underestimating the risk, whereas given the scope of the forecast sample only $1750 \cdot 0.01 = 17.5 \approx 18$ erroneous estimates are expected.

The results show that, in case of stock indexes, traditional approaches to VaR estimates mostly substantially underestimate the actual risk. In particular, the variable-co-variable (VC) method works relatively well only at the 95% VaR level, with 99% of the level failing – more specifically, the VC of all methods used most significantly underestimates the real risk of providing the US stock index 3.1 times than expected. The paramount model used in RiskMetrics with EWMA (RM) application fails, and so does the VC method, in being applied within 99% reliability; on the other hand, it provides better output than VC – which is probably due to the assumed conditional volatility. From traditional approaches historical simulation provides the best results. As for the difference between the 250-sample and the 500-sample versions, the first one seems to be more accurate than the larger 500-sample version, although the differences between the outputs provided are not significant. Figure 3 included in the appendix represents the development of one-day 99% of VaR estimates for each serie. Graphs on the left side represent traditional approaches. Comparing both versions of the historical simulation, it is clear from the figures that the 250-sample version tends to overstate the risk, but adapts more quickly to the changing market volatility.

Table 2: Ratios *â/α* for 1-day VaR estimates – traditional approaches

	0,05		0,01		0,05		0,01	
	SP	DJES	SP	DJES	USD	JPY	USD	JPY
VC	1.1314	1.1200	**3.0857**	**2.5143**	0.9143	1.0871	**1.9429**	**2.0000**
EWMA	1.2571	1.2571	**2.5714**	**1.8857**	1.0057	1.1657	1.3143	**1.8286**
HS250	1.0171	1.0982	**1.4286**	**1.5429**	1.0171	1.0857	**1.5429**	1.1429
HS500	1.0514	**1.6914**	**1.4857**	**1.7714**	0.9600	1.0171	0.0571	1.0286

Source: Author's own processing

Table 3 represents the mentioned ratios in case of application of GARCH class models within a parametric approach. The results show that in all cases the deviations between *â* and *α* are statistically insignificant, i.e., their values can be considered identical. Furthermore, it is clear that estimates made using GARCH models are more accurate than outputs from traditional methods. The most accurate results are framed in bold for each model, and it is clear that the highest precision is not given directly by the particular model but by the distribution of standardized residues.

Table 3: Ratios *â/α* for 1-day VaR estimates – GARCH models

	0,05		0,01		0,05		0,01	
	SP	DJES	SP	DJES	USD	JPY	USD	JPY
GARCH-n	0,9486	1,0623	0,8571	1,1804	0,8254	0,8147	0,9648	0,8576
GJR-G-n	0,9486	1,1314	0,8571	1,2571	0,8800	0,8686	1,0286	0,9143
TGARCH-n	0,9338	1,1314	0,8438	1,2571	0,8595	0,8484	1,0047	0,8930
GARCH-t	0,8199	0,9854	0,7551	1,0667	0,8770	0,8136	1,0038	0,7925
GJR-G-t	0,8686	1,1086	0,8000	1,2000	0,9486	0,8800	1,0857	0,8571
TGARCH-t	0,8523	1,0896	0,7850	1,1795	0,9217	0,8551	1,0550	0,8329
GARCH-st	1,0406	1,1938	0,9584	1,2923	1,1422	1,0596	1,3073	1,0321
GJR-G-st	1,0406	1,2258	0,9584	1,3269	1,1712	1,0865	1,3405	1,0583
TGARCH-st	1,0394	1,2685	1,0218	1,3731	1,1712	1,0865	1,3405	1,0583

Source: Author's own processing

Indeed, it can be seen from table 3 that the differences between GARCH models are primarily determined by selecting the z_t division. Models with the same distribution, but with different approaches to volatility, have similar *â/α* ratios than models with the same access to volatility and different distributions. In case of the US stock index, the best results are achieved by using the titled Student's distribution, both at lower and higher levels of reliability. In case of the European Index, the Student's distribution appears to be the best choice. For the estimate of VaR returns of exchange rate, the use of tilted Student's distribution seems best for JPY/EUR, in case of USD/EUR then for 95% of the Student distribution VaR application. The highest accuracy of the 99% VaR of the USD/EUR exchange rate appears to be achieved by using a normal distribution, but the application of the Student's distribution also generates good outputs. Overall, therefore, the application of the conditional volatility models used generates very similar, if not in some cases identical results. Differences are created by choosing specific types of division. When choosing a specific approach for VaR estimation, it is necessary to focus primarily on the probability distribution of the analysed returns, or more precisely their residuals, with the most accurate estimates being provided by Student's distribution, or more precisely tilted Student's distribution.

Graphs on the right-hand side of figure 3 represent the development of VaR estimates made by applying GJR-GARCH models with normal Student's distribution. Tilted Student's distribution is not included in the charts for clarity. Furthermore, these graphs present the estimates made using RM access and the GJR-GARCH-EVT model with Student distribution. The graphs show that the RM application has the highest tendency to underestimate the risk – it provides the lowest VaR values. Similarly low values are obtained by application of a tilted Student's distribution. On the contrary, GJR-GARCH-EVT application, which has a strong tendency to overestimate the real risk, provides the highest estimates. This feature is also characteristic for GARCH-EVT and TGARCH-EVT, as shown in table 4. Again, similar performance of models is confirmed using the same distribution – in this case the Student's distribution within conditional volatility models and the generalized Pareto division within the EVT.

Table 4: Ratios *â/α* for 1-day VaR estimates – GARCH-EVT models

	0,05		0,01		0,05		0,01	
	SP	DJES	SP	DJES	USD	JPY	USD	JPY
GARCH-evt	0,8486	0,8758	0,6223	1,1640	0,7952	0,7280	0,7840	0,7280
GJR-G-evt	0,8571	0,9029	0,6286	1,2000	0,8114	0,7429	0,8000	0,7429
TGARCH-evt	0,8829	0,9390	0,6474	1,2480	0,8277	0,7577	0,8160	0,7577

Source: Author's own processing

Table 5 for each of the tested VaR models includes for each VaR estimate average values and standard deviations of the observed ratios. The standard deviation in this case determines the rate of deviation of a given ratio from the value 1, i.e., for the standard deviation SD_i, belonging to the i-model, it applies that

$$SD_i = \frac{1}{N}\sqrt{\sum_{j=1}^{N}\left[(\hat{\alpha}/\alpha)_j - 1\right]^2}, \tag{16}$$

where *N* denotes the number of the valuated ranks (within a certain degree of reliability). In our case, $N = 4$. The models with the smallest standard deviation tend to be consistent across the series of returns. For each row and reliability level, an average closest to one and the deviation closest to zero is selected. In terms of the average ratios valuation for traditional approaches to VaR estimation, the best performance is provided by nonparametric historical simulation, namely its 250-sample version, which shows the lowest deviation of the observed estimates.

Table 5: Averages and deviations of ratios *â/α* – traditional approaches

	0,05		0,01	
	Average	SD	Average	SD
VC	1,0632	0,1080	2,3857	1,4605
RM	1,1714	0,1998	1,9000	1,0049
HS250	1,0514	0,0618	1,4143	0,4454
HS500	1,1800	0,3474	1,0857	0,6559

Source: Author's own processing

In terms of GARCH models, the application of the Student's and standard distribution provide similar deviations between estimates made by them. The same is true about average estimates. On the contrary, the application of the tilted Student's distribution shows higher deviations in the estimates and thus the higher uncertainty of the estimates obtained. If we compare the results for GARCH models with historical simulation (best rated traditional approach), then it is obvious that HS shows a significantly higher uncertainty of realized estimates than is the case for GARCH models. The uncertainty of the HS is even higher than that of the GARCH-EVT models – see table 7.

Table 6: Averages and deviations of averages *â/α* – GARCH models

	0,05		0,01	
	Average	SD	Average	SD
GARCH-n	0,9128	0,1335	0,9650	0,1364
GJR-G-n	0,9571	0,1136	1,0143	0,1539
TGARCH-n	0,9433	0,1269	0,9996	0,1597
GARCH-t	0,8740	0,1437	0,9045	0,1639
GJR-G-t	0,9514	0,1074	0,9857	0,1641
TGARCH-t	0,9297	0,1193	0,9631	0,1654
GARCH-st	1,1090	0,1255	1,1475	0,2137
GJR-G-st	1,1310	0,1495	1,1710	0,2387
TGARCH-st	1,1414	0,1661	1,1984	0,2545

Source: Author's own processing

Table 7: Averages and deviations of averages *â/α* – GARCH-EVT models

	0,05		0,01	
	Average	SD	Average	SD
GARCH-evt	0,8119	0,1964	0,8246	0,2694
GJR-G-evt	0,8286	0,1813	0,8429	0,2665
TGARCH-evt	0,8518	0,1627	0,8673	0,2638

Source: Author's own processing

For a better idea of the quality of the tested models, we will further focus on the results of the CC test, which unlike the UC test provides a more comprehensive way of back testing. For this purpose, table 8 presents the number of failures of the VaR estimates followed in terms of the statistical significance of UC and CC tests. If we consider the maximum number of failures across the tests as a test quality indicator, then the results point to a clear preference of approaches with the application of the conditional volatility model. Specifically, at a lower level of reliability, the parametric approach with the application of the GJR-GARCH model with Student residue distribution shows the best performance. Within the 99% reliability level, the GJR-GARCH application approaches results are similar.

Table 8: Results of backwards testing

	0,05		0,01	
	UC	CC	UC	CC
VC	0	3	4	1
EWMA	0	3	3	2
HS250	0	3	3	3
HS500	1	3	2	3
GARCH-n	0	2	0	2
GARCH-t	0	0	0	2
GARCH-st	0	1	0	3
GARCH-evt	0	2	0	3
GJR-G-n	0	2	0	2
GJR-G-t	0	0	0	2
GJR-G-st	0	1	0	2
GJR-G-evt	0	2	0	3
TGARCH-n	0	2	0	3
TGARCH-t	0	0	0	2
TGARCH-st	0	1	0	3
TGARCH-evt	0	1	0	3

Source: Author's own processing

Discussion of results

In this part, the performance of the connection of conditional volatility models to the parametric and semi-parametric approach to the VaR estimation was verified. The results ascertained show that the application of these models tends to provide more accurate predictions of actual losses compared to both the parametric approach based on RiskMetrics and traditional approaches such as the variation-covariance method or historical simulation. Overall, the application of conditional volatility models ensures that VaR estimates are more flexible to adapt to changing market conditions – especially in the periods associated with higher return volatility. The results also show that the differences between individual models the conditional volatility are primarily determined by selecting the specific distributions of the standardized return series.

In terms of predictive performance testing of individual models of conditional volatility, then models with the same distribution, but with different approaches to volatility, have similar features, unlike models with the same approach to volatility and different residue distributions. Thus, we are finding out that choosing a particular probability distribution is far more important than choosing the model itself. It is clear from the analysis conducted that there is a very good student distribution across the various risk market environments. If we compare the results obtained with the previously published works, for example, Bams et al. (2005); Hartz et al. (2006), So and Yu (2006), Mutu et al (2011), Abad et al. (2016) show a significant contribution of conditional volatility models to the accuracy of VaR or possibly ES estimates. Furthermore, the above-mentioned works did not primarily address the relationship between a particular model of conditional volatility and a specific distribution of returns. However, by a more detailed analysis of these texts we can trace a more pronounced predominance of positively evaluated approaches involving different models of conditional volatility with the application of the Student's distribution.

Conclusion

The aim of this paper is to determine the importance of choosing the conditional volatility model when estimating VaR. Specifically; the assumption was tested of whether the involvement of models taking into account the risk-adjusted return volatility model always refines the VaR estimates. Furthermore, whether the choice of the suitable type of revenue distribution is as important as choosing a suitable conditional volatility model. These assumptions have been verified through empirical research consisting of data analysis, namely stock index returns and exchange rates. Data was collected from publicly available sources.

It is clear from the results that the VaR estimates obtained in the context of the application of conditional volatility models are far more flexible to adapt to changing market conditions than traditional approaches. The results also show that the differences between individual models of contingent volatility are primarily attributable to the selection of specific residue distributions. In terms of predictive performance testing of conditional volatility models, models with the same distribution, but with different approaches to volatility, have similar features, unlike models with the same approach to volatility and different residue distributions.

Overall, the research carried out confirms the significance of the parametric and semi-parametric approach in measuring market risk. These approaches are not yet widely used by financial institutions. The research carried out complements these approaches with some innovative elements, more precisely; it has some characteristic features that make both parametric and semi-parametric approaches more accurate VaR estimates than traditional approaches.

Acknowledgements

This article was created at the University of Finance and Administration within the project titled "New Opportunities and Approaches to Measuring and Managing Market Risks", no. 7427/2017/05 supported by means of targeted support for specific university research.

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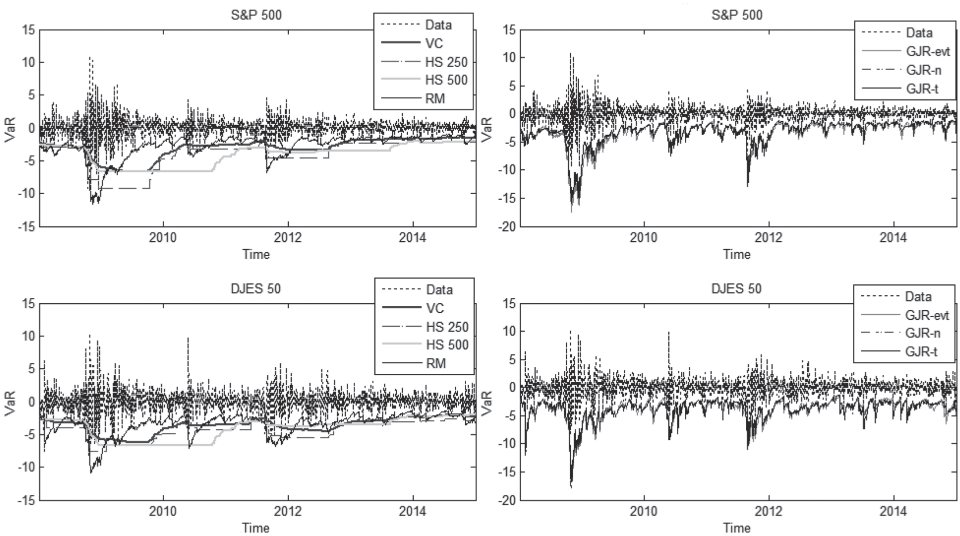
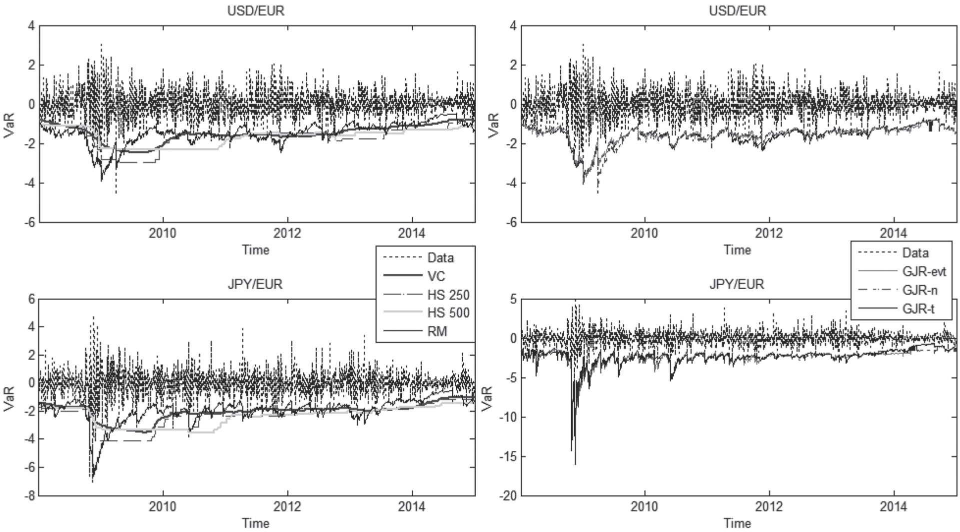
Appendix

Figure 3: 1-day 99% estimates of VaR returns of stock indexes and currency exchange rates





Source: Author's own processing

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